MEASUREMENT of VOLUME and FLOW

Lecture Notes

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Essential measurement for the assessment of the physiological condition of a patient.

1. O_2 concentration and other nutrients in the blood $\longrightarrow$ First class but difficult to apply.
2. Blood flow measurement $\longrightarrow$ Second class but easier to apply.
3. Blood pressure and ECG $\longrightarrow$ Third class but totally noninvasive.

Cardiac Output

It is the quantity of blood delivered by the heart to the aorta per minute. It is a major determinant of oxygen delivery to the tissues.

Indicator Dilution Method for Measuring Cardiac Output

Principle : Introducing into or removing from a stream of fluid a known amount of indicator and measuring the concentration difference upstream and downstream of the injection site to estimate the volume flow of the fluid.

Concentration of indicator with mass m_0 added to a volume V is $C = \frac{m_0}{V}$

Incremental change of C is $\Delta C = \frac{m}{V}$

To maintain a fixed amount of change in C , a fixed quantity of indicator per time $\frac{dm}{dt}$ must be continuously added:

The rate of indicator leaving the vessel :

$C_0 F = C_i F + \frac{dm}{dt} \longrightarrow (C_0 - C_i)F = \frac{dm}{dt}$

or $F = \frac{dm/dt}{C_o - C_i}$

Fick Technique

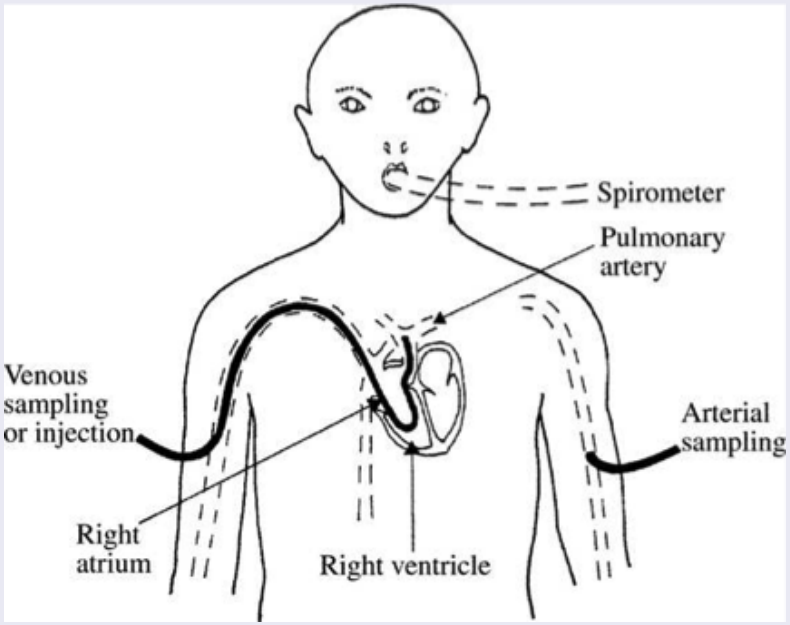
$$\text{Cardiac Output} : F = \frac{dm/dt}{C_a - C_v}$$

F : blood flow, liters/min

dm/dt : consumption of O_2 liters/liter

C_a : arterial concentration of O_2, liters/liter

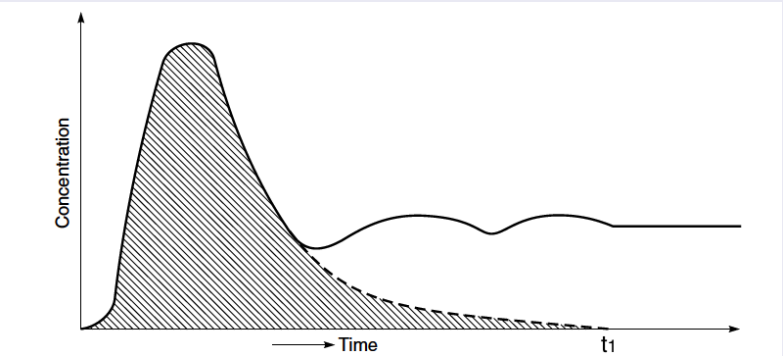
C_v : venous concentration of O_2, liters/liter



Returning blood measured from pulmonary artery after the blood from the upper and lower body pumped by right ventricle.

- Oxygen as an indicator is breathed in through the spirometer.
- The exhaled CO_2 is captured by soda-lime canister.
- The arterial C_a and venous C_v blood oxygen concentrations are measured with a catheter.
- Consumption of oxygen is indicated directly determining the net gas flow.

Rapid Injection Indicator Dilution Method



After the bolus is injected, it would normally follow the dashed curve. Because of recirculation, it follows the bold one.

An increment of blood volume passing the site in time dt contains an amount of indicator :

$$dm = C(t)dV \longrightarrow \frac{dm}{dt} = C(t)\frac{dV}{dt} = C(t)F(t)$$

Integrating over time,

$$m = \int_0^{t_1} C(t)F(t)dt$$

Assuming that flow is almost constant over time,

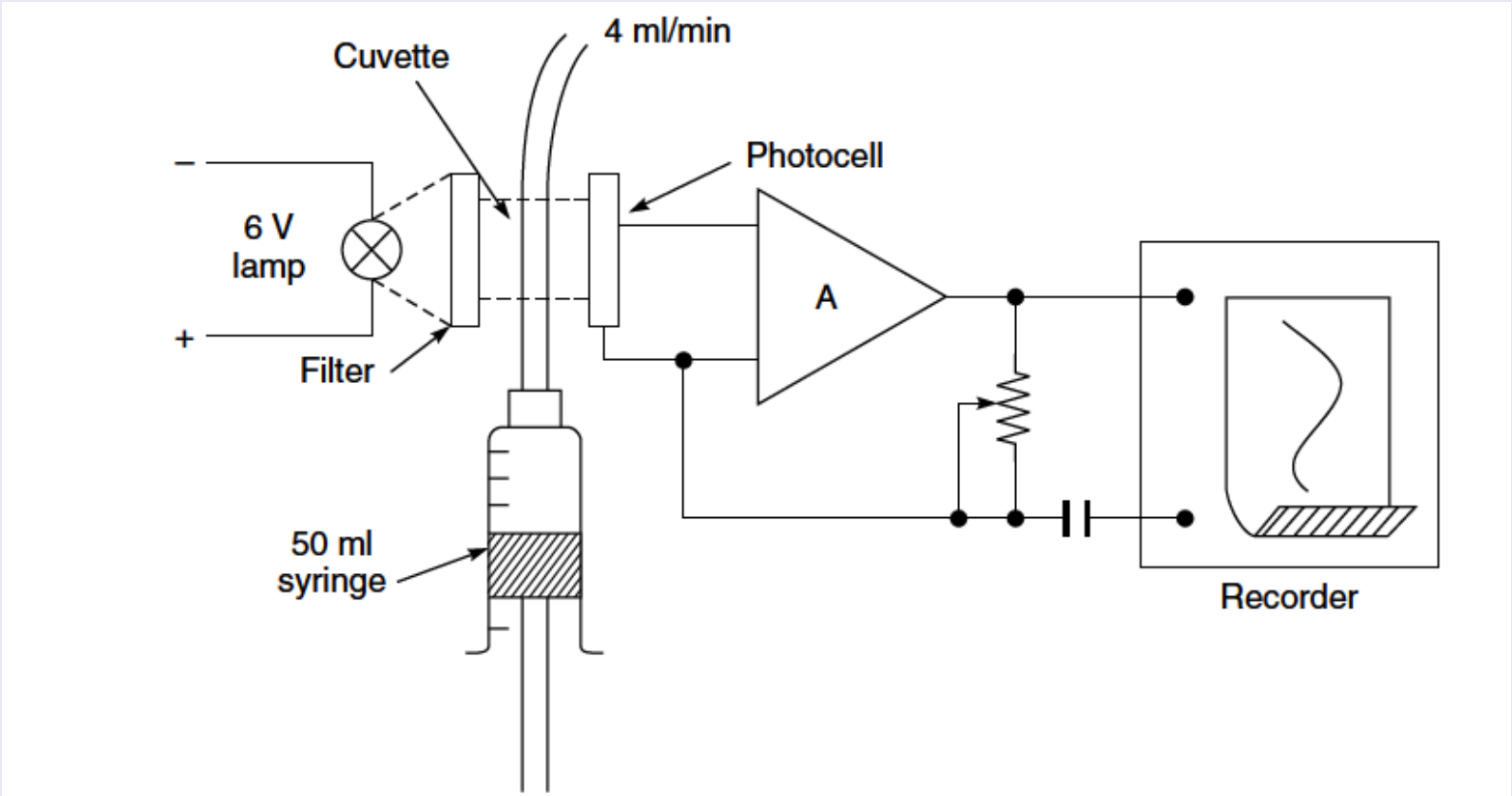
The average flow is $F = \frac{m}{\int_0^{t_1} C(t)dt} = m/Shaded\ Area$

A computer extrapolates the dashed curve and calculate the area.

Dye Dilution Method

- *Indocyanine Green* : i) Inert, ii) harmless, iii) measurable, iv) economical and v) absorbing light in the 800 nm which is common to both reduced and oxygenated hemoglobin.
- Injecting the dye (5 mg/ml) into the right atrium by means of a venous catheter.
- Blood is drawn blood from the radial or femoral artery through a cuvette.
- The curve is measured by a densitometer.
- After the curve is drawn, an injection of saline is given to flush out the dye from the circulating blood.
- For optimum accuracy, the amount of injected dye should yield a peak concentration less than 20 mg/ml in the dilution curve.

Densitometer to Measure Dye Concentration



Two sources of distortion:

1. Nonuniform flow velocity of the fluid in the catheter causing the dye to mix.
2. Slower response characteristics of measuring system to record instantaneous dye concentration.

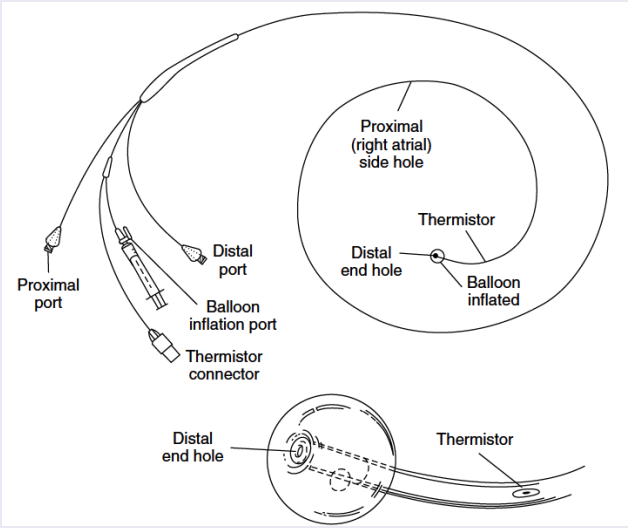
Thermal Dilution Method

A thermal indicator of known volume introduced into either the right or left atrium will produce a resultant temperature change in the pulmonary artery or in the aorta respectively, the integral of which is inversely proportional to the cardiac output. Blood temperature is measured over a range of 30 to 40^{o}C with an accuracy of $\pm 0.2^o$C in the range 0-1^{o}C full scale.

Cardiac Output :

$$F = \frac{Q}{\rho_b C_b \int_0^{t_1} \Delta T_b(t) dt} \, m^3/s$$

Q : heat content of injectate (J),
$= K \Delta T$
ρ_b : density of blood
C_b : specific heat of blood J/(kg·K)



A 4-lumen catheter
Distal Lumen: connects to transducer system to measure
(i) pulmonary artery pressure
(ii) Wedge pressure with balloon inflated.
Inflation Lumen connects to balloon located approximately 1 mm from catheter-tip.
Proximal lumen to monitor central venous pressure or right atrium pressure.
Thermistor Lumen connects to computer.

Continuous Measurement of Cardiac Output from Aortic Pressure

During ejection of blood into aorta incremental amount of blood is

$$dV = \frac{1}{Z_{ao}}(P_{ao} - P_{ed})dt$$

$$A = \int_{T_O}^{T_E} (P_{ao} - P_{ed})dt$$

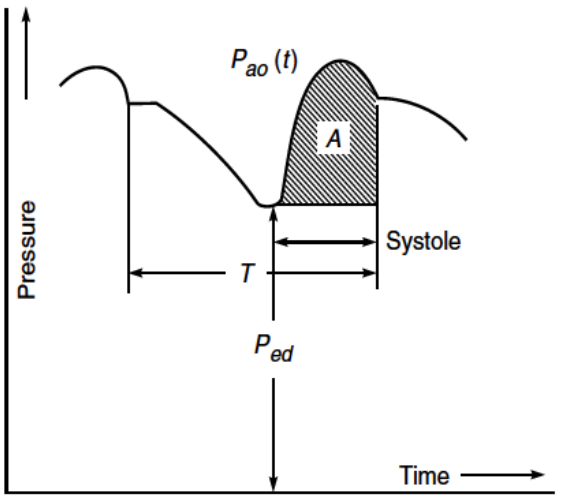
Z_{ao} : aortic impedance

$$Stroke\ Volume = \frac{A}{Z_{ao}}$$

Cardiac Output

$$= HR \times Stroke\ Volume$$

$$= \frac{60 \cdot A}{T \cdot Z_{ao}}\ \text{cm}^3$$



Electromagnetic Flowmeter

Voltage due to blood flow in a magnetic field (Faraday's law);

$$v = \int_0^L \mathbf{u} \times \mathbf{B} \cdot d\mathbf{L} = BLu$$

L : length between electrodes

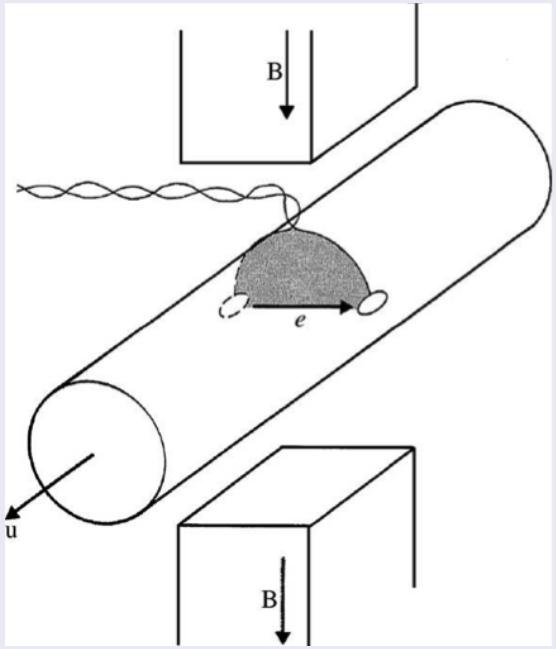
B : Magnetic flux density

u : instantaneous speed of blood

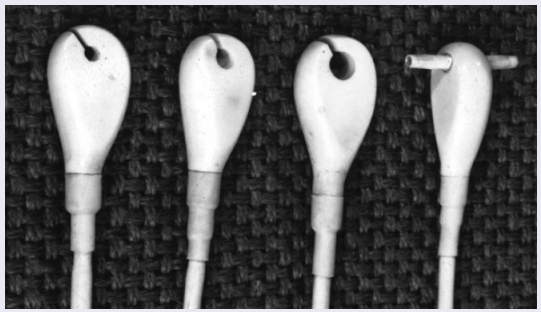
If the flow rate $F = uA$

v is proporitonal to flow

$v = \frac{BL}{A}F$



Magnetic probes for blood vessels with various sizes



Average flow rate :

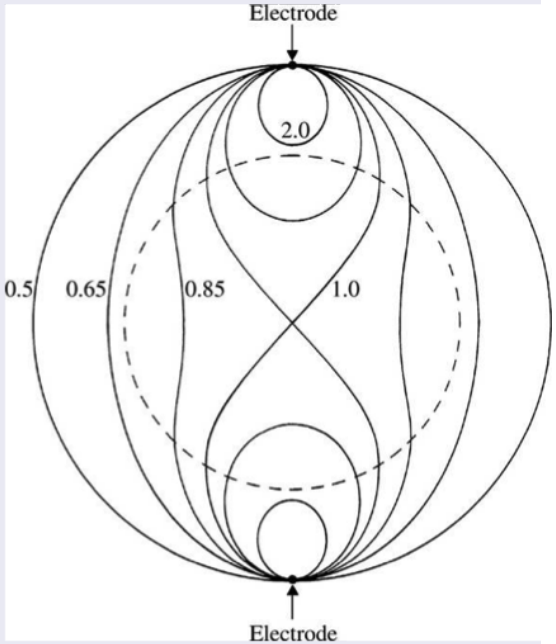
20-25 cm/s in arteries

10-12 cm/s in veins

Factors Causing Errors due to Asymmetric Velocity Profile

i) Regions of high velocity generate higher currents than regions of low velocity causing voltage drop in transverse plane.
ii) Shunting effect of the wall due to the variation in the conductivity ratio of the wall to the blood.
iii) Shunting effect on the flow signal due to conductivity difference between the wall and its outside.
iv) Non uniform magnetic flux denisty in the transverse plane causing additional currents circulating.
v) Non uniform magnetic flux along the flow axis.

Weight function for velocity constributions



Alternating Current Flowmeters

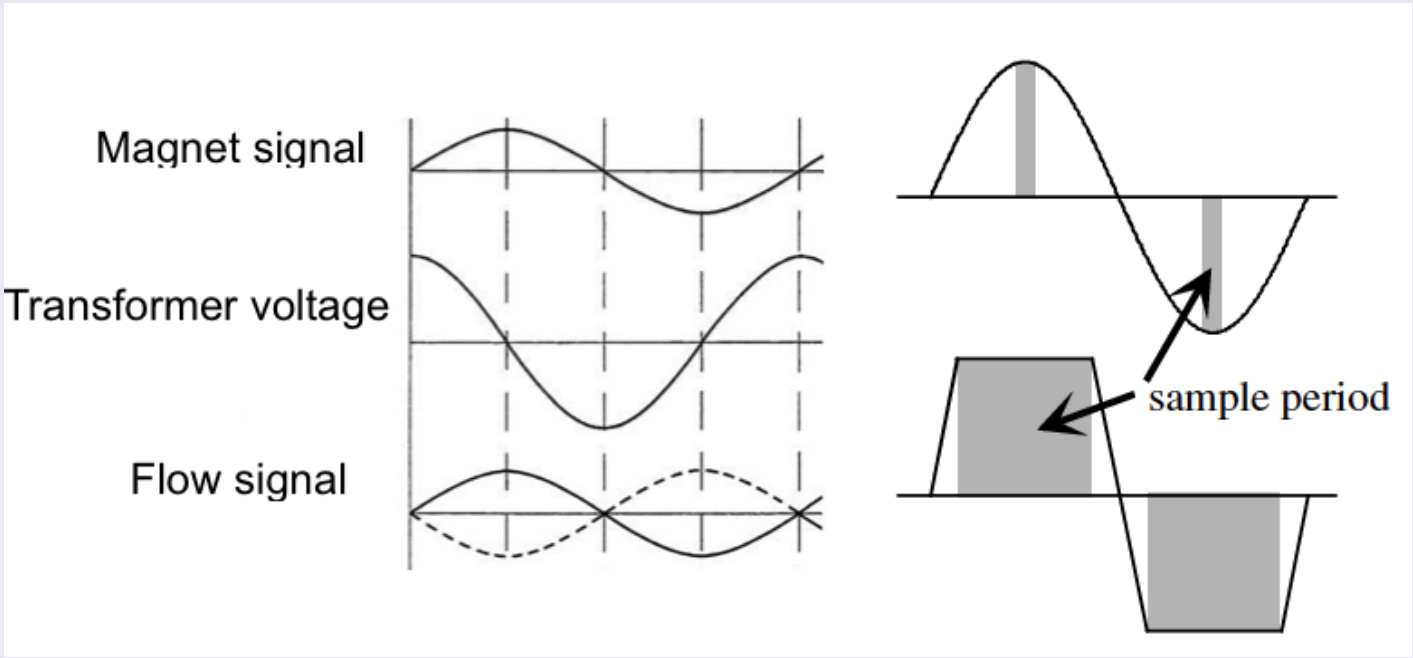
For a sinusoidal magnetic field $B \sin \omega t$, the induced voltage is

$$v = BL(u \sin \omega t + k \cos \omega t)$$

$k \cos \omega t = \frac{d}{dt} \sin \omega t$ is the induced transformer voltage due to the changing magnetic field if the loop is not parallel to the field.

Solution I:

Gated Signal Sampling

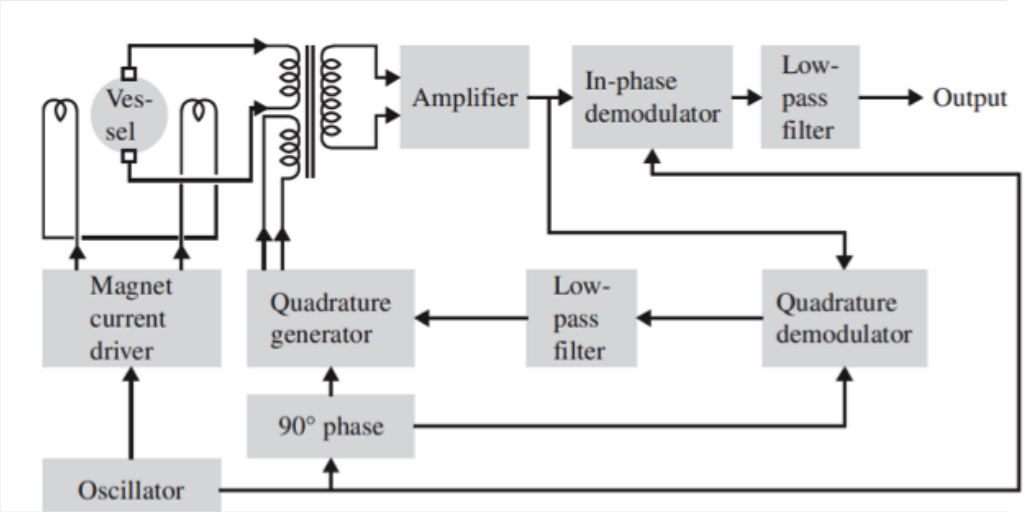


Solution II:

Quadrature demodulator performs detection and full wave rectification.

Low pass filter yields a DC voltage that corresponds to magnitude of transformer voltage.

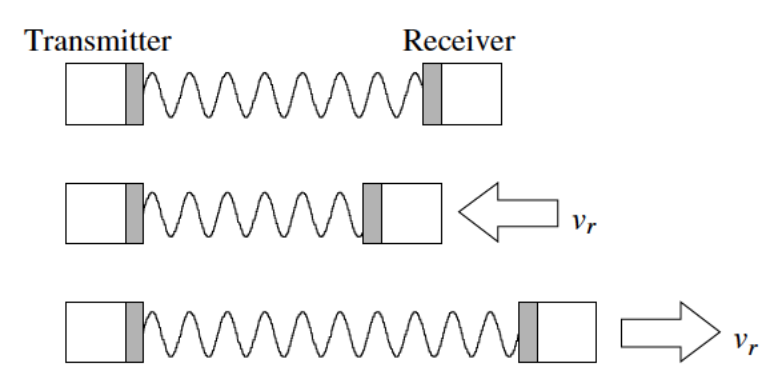
DC voltage is quadrature modulated and fed into balancing coil on the input transformer as a negative feedback.

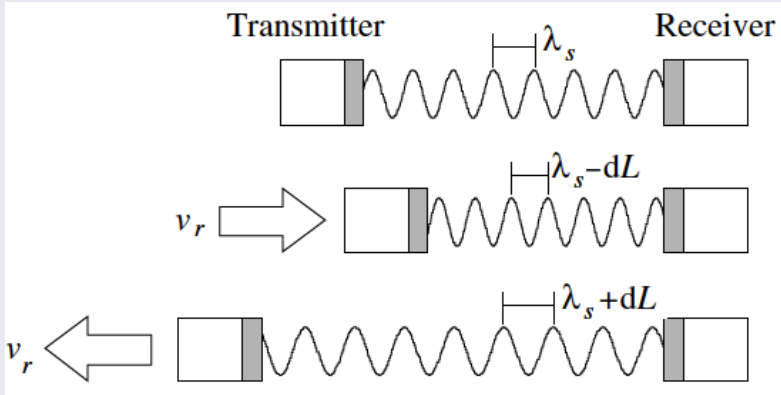


Doppler Effect

In unit time, the receiver moves a distance v_r and detects an additional number of peaks $\frac{v_r}{\lambda_s}$.
The number of peaks detected per time $f_r = f_s + \frac{v_r}{\lambda_s}$
$f_r = f_s + \frac{v_r}{c} f_s$
Doppler frequency : $f_D = \frac{v_r}{c} f_s$

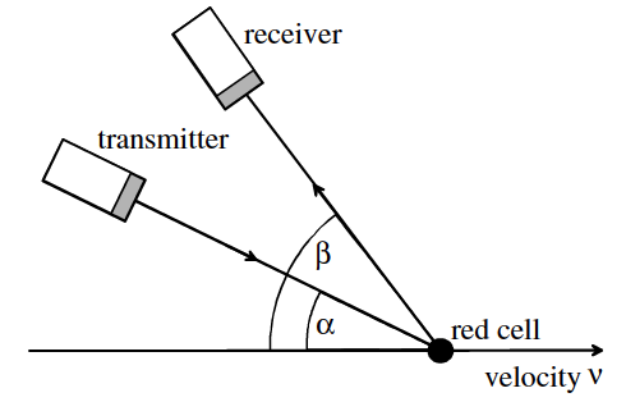
For a moving source towards receiver, the time interval between peaks is $1/f_s$.
In this time, source moves a distance $dL = \frac{v_s}{f_s}$ where v_s is source velocity.
The wavelength λ_r at the receiver is
$\lambda_r = \lambda_s - dL = \frac{c}{f_s} - \frac{v_s}{f_s} = \frac{c}{f_r}$.
$f_r = \frac{c}{c - v_s} f_s = \frac{1}{1 - \frac{v_s}{c}} f_s \approx (1 + \frac{v_s}{c}) f_s$





Doppler Effect

The frequency seen by the moving particle is $\omega_p = \omega_c + (v_p \cos\alpha/c)\omega_c$ where ω_c is transmitter frequency and ω_p is radiated fequency from moving particle.
The frequency at receiver is
$\omega_r = (1 + v_p \cos\beta/c)\omega_p$
$= (1 + v_p \cos\beta/c)(1 + v_p \cos\alpha/c)\omega_c.$
$\approx \omega_c + v_p/c(\cos\alpha + \cos\beta)\omega_c$



Doppler frequency $\omega_D = v_p/c(\cos\alpha + \cos\beta)\omega_c$

If $\phi = \alpha - \beta$ and $\theta = (\alpha + \beta)/2$ then

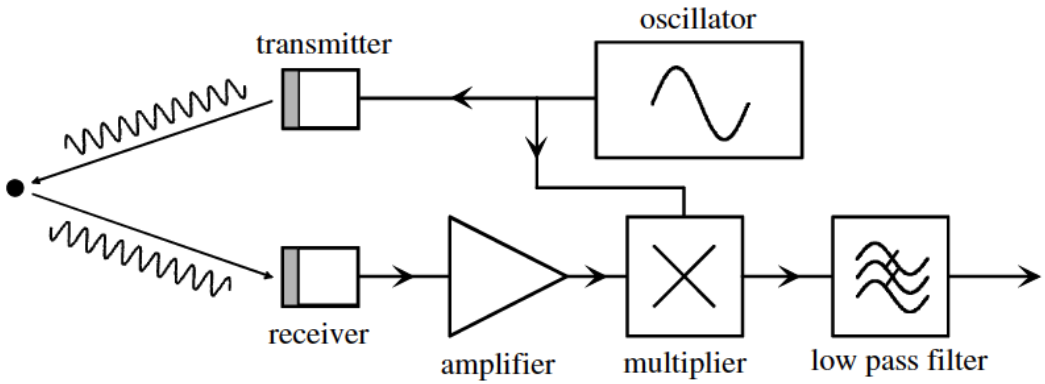
$$\omega_D = \frac{2v_p\omega_c}{c}\cos\theta\cos(\phi/2)$$

The Doppler frequency is proportional to flow velocity.

If $\phi = 0$ then

$$\omega_D = \frac{2v_p\omega_c}{c}\cos\theta$$

Transmitted signal :
$E \cos \omega_c t$
Received signal :
$V_i = A \cos(\omega_c t + \phi)$
$+B \cos(\omega_c + \omega_D)$
ϕ is the phase shift
in carrier component.



Doppler component is obtained from the second term by demodulation.

$V_A = AE \cos(\omega_c t + \phi) \cos(\omega_c t) + BE \cos(\omega_c + \omega_D)t \cos(\omega_c t)$
$= \frac{AE}{2} [\cos(2\omega_c t + \phi) + \cos(\phi)] + \frac{BE}{2} [\cos(2\omega_c t + \omega_D t) + \cos(\omega_D t)]$

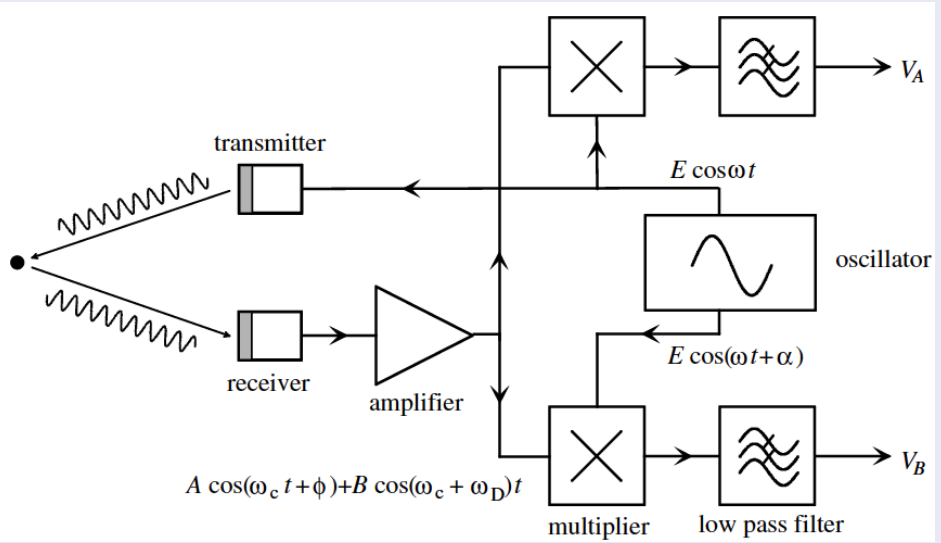
$\cos(\phi)$ is a DC signal and removed by high pass filtering.

Since $\omega_D << \omega_c$, the $2\omega_c$ frequency signals are eliminated by low pass filtering.

Doppler signal is $\cos(\omega_D t)$ with a directional ambiguity of the flow direction since $\cos(\omega_D t) = \cos(-\omega_D t)$.

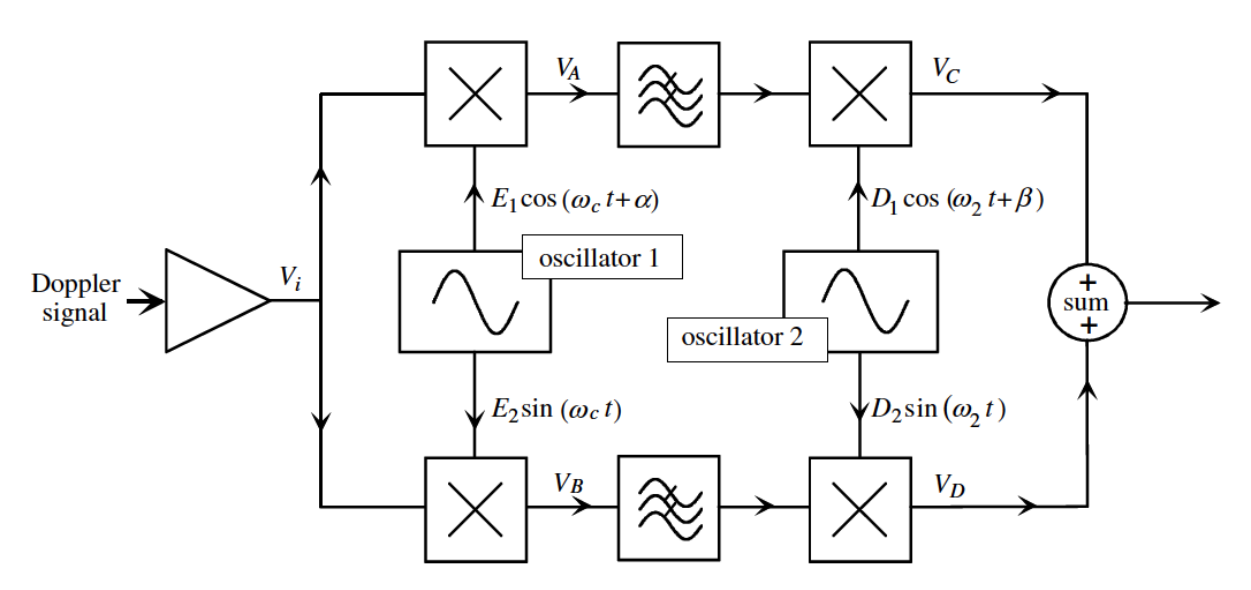
Flowmeter with Directional Demodulator

Two signals are used.
$(A\cos(\omega_c t + \phi) +$
$B\cos(\omega_c + \omega_D)t)$
$\times\ E\cos(\omega_c t)$ which is
$V_A = \frac{1}{2}BE\cos(\omega_D t)$
$V_B = \frac{1}{2}BE\cos(\omega_D t - \alpha)$
after removing DC
and high frequencies.



For $\omega_D > 0$, phase difference between V_B and V_A is ϕ.
For $\omega_D < 0$, phase difference between V_B and V_A is $-\phi$.

Flowmeter with Directional Demodulator



Positive

Doppler frequencies are added to ω_2, and negative Doppler frequencies are subtracted from ω_2.

The outputs of the oscillators should have a phase difference of $\pi/2$.

α and β represent the error in the phase difference.

$V_i = A\cos(\omega_c t + \phi) + B\cos(\omega_c + \omega_D)t$

$V_A = \frac{BE_1}{2}\cos(\omega_D t - \alpha)$ and $V_B = \frac{BE_2}{2}\sin(\omega_D t)$

$V_C = \frac{BE_1 D_1}{2}\cos(\omega_D t - \alpha)\cos(\omega_2 t + \beta) =$

$\frac{BE_1 D_1}{4}[\cos(\omega_D t + \omega_2 t - \alpha + \beta) + \cos(\omega_D t - \omega_2 t - \alpha - \beta)]$

$V_D = -\frac{BE_2 D_2}{4}\sin(\omega_D t)\sin(\omega_2 t) = \frac{BE_2 D_2}{4}[\cos(\omega_D - \omega_2)t - \cos(\omega_D + \omega_2)t]$

$$V_C + V_D =$$
$$\frac{B}{4}\left[E_1 D_1 \cos((\omega_D + \omega_2)t - \alpha + \beta) + E_2 D_2 \cos((\omega_D + \omega_2)t)\right] +$$
$$\frac{B}{4}\left[E_1 D_1 \cos((\omega_D - \omega_2)t - \alpha - \beta) + E_2 D_2 \cos((\omega_D - \omega_2)t)\right]$$

$$USB = \frac{B}{4}\left[(E_1 D_1)^2 + (E_2 D_2)^2 + 2(E_1 D_1)(E_2 D_2)\cos(\beta - \alpha)^{1/2}\right]$$
$$\sin\left[(\omega_D + \omega_2)t - \tan^{-1}\left(\frac{E_1 D_1 + E_2 D_2 \cos(\beta - \alpha)}{E_2 D_2 \sin(\beta - \alpha)}\right)\right]$$

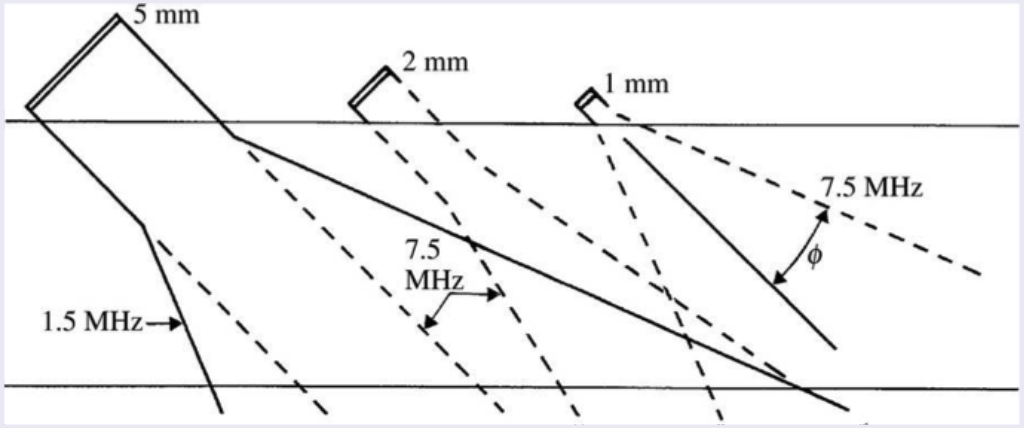
$$LSB = \frac{B}{4}\left[(E_1 D_1)^2 + (E_2 D_2)^2 - 2(E_1 D_1)(E_2 D_2)\cos(-\beta - \alpha)^{1/2}\right]$$
$$\sin\left[(\omega_D - \omega_2)t + \tan^{-1}\left(\frac{E_1 D_1 - E_2 D_2 \cos(-\beta - \alpha)}{E_2 D_2 \sin(-\beta - \alpha)}\right)\right]$$

The USB signal has a frequency $\omega_D + \omega_2$.

Forward flow gives a higher frequency than ω_2 and a reverse one, gives a lower frequency.

Ultrasonic Transducers

Near field $d_{nf} = \frac{D^2}{4\lambda}$
D : transducer diamater
λ : wavelength



In the near field, there is spreading but a nonuniform intensity due to diffraction casued by interference.
In the far field, the beam diverges with its intensity inversely proportional to the square of the distance and a divergence angle

$$\sin\phi = \frac{1.2\lambda}{D}$$

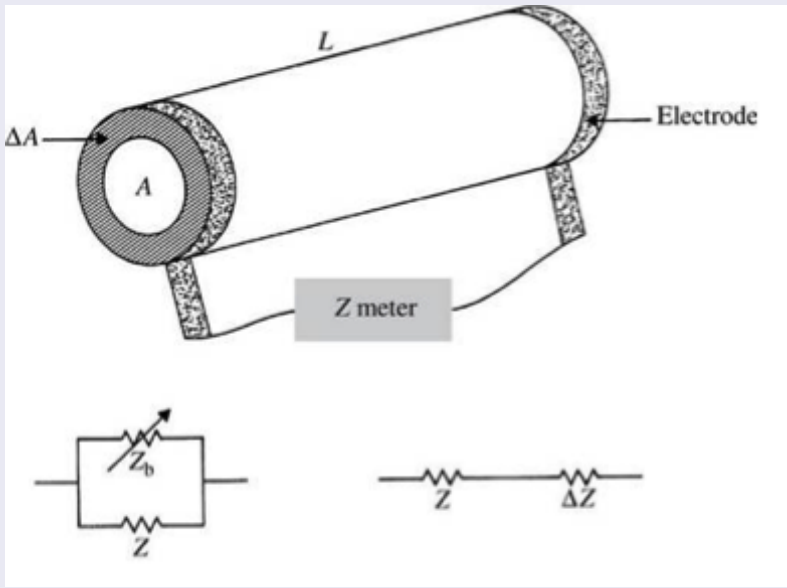
To achieve good resolution near field is used which favors high frequencies and larger transducers.
Absorbtion of heat by the tissue is proportional to frequency which suggests a low frequency operation.
Compromise is between $2 - 10$ MHz.

Electrical Impedance Plethysmography

Cylindrical limb model
L : length
A : cross-sectional area
Z_b : impedance of blood

With each pressure pulse,
A increases by ΔA,
Z_b added in parallel to Z
ΔZ is measured instead of Z_b.

A model for impedance plethysmography



$$Z_b = \frac{\rho_b L}{\Delta A}$$

$$\Delta V = L\Delta A = \frac{\rho_b L^2}{Z_b}$$

$$\Delta Z = Z_b||Z - Z = \frac{ZZ_b}{Z+Z_b} - Z = -\frac{Z^2}{Z+Z_b} \approx -\frac{Z^2}{Z_b} \text{ since } Z_b >> Z$$

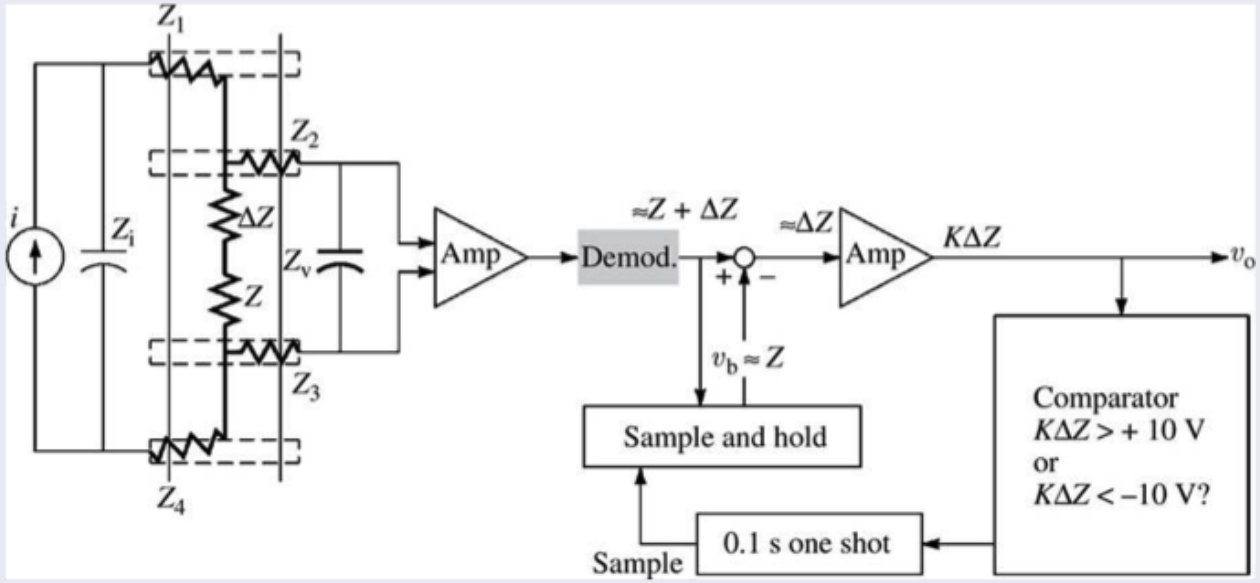
$$\Delta V = -\frac{\rho_b L^2 \Delta Z}{Z^2}$$

Four-electrode Impedance Plethysmography

Z_i : stray capacitance

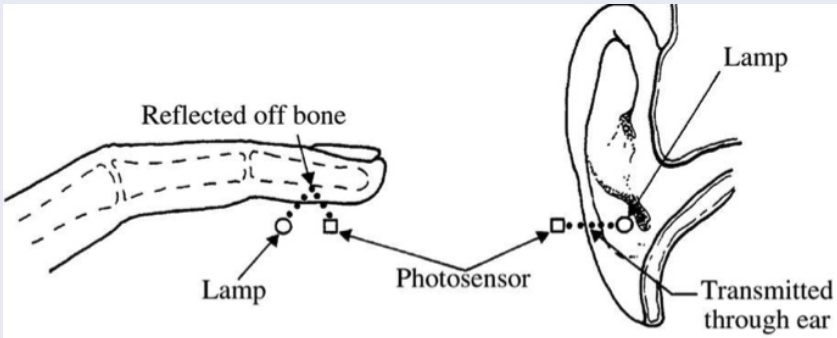
Z_1 to Z_4 : electrode impedances

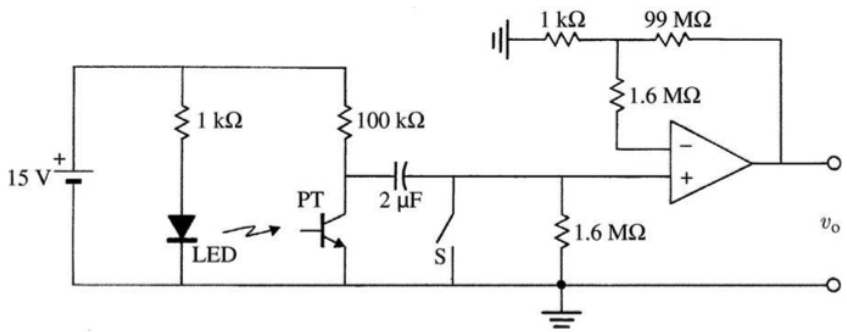
ΔZ modulates input current



Constant current (>1mA and 20-100 KHz) is injected through two outer electrodes, and voltage is sensed between two inner electrodes.
Amplification and demodulation yield $Z + \Delta Z$.
A balancing voltage v_b is applied to produce the desired ΔZ.
If saturation of v_o occurs, the comparator commands the sample and hold to sample $Z + \Delta Z$ and hold it as v_b.
This resets the input to the final amplifier and v_o zero.
Further changes in ΔZ cause changes in v_o without saturation.

Optical Plethysmography



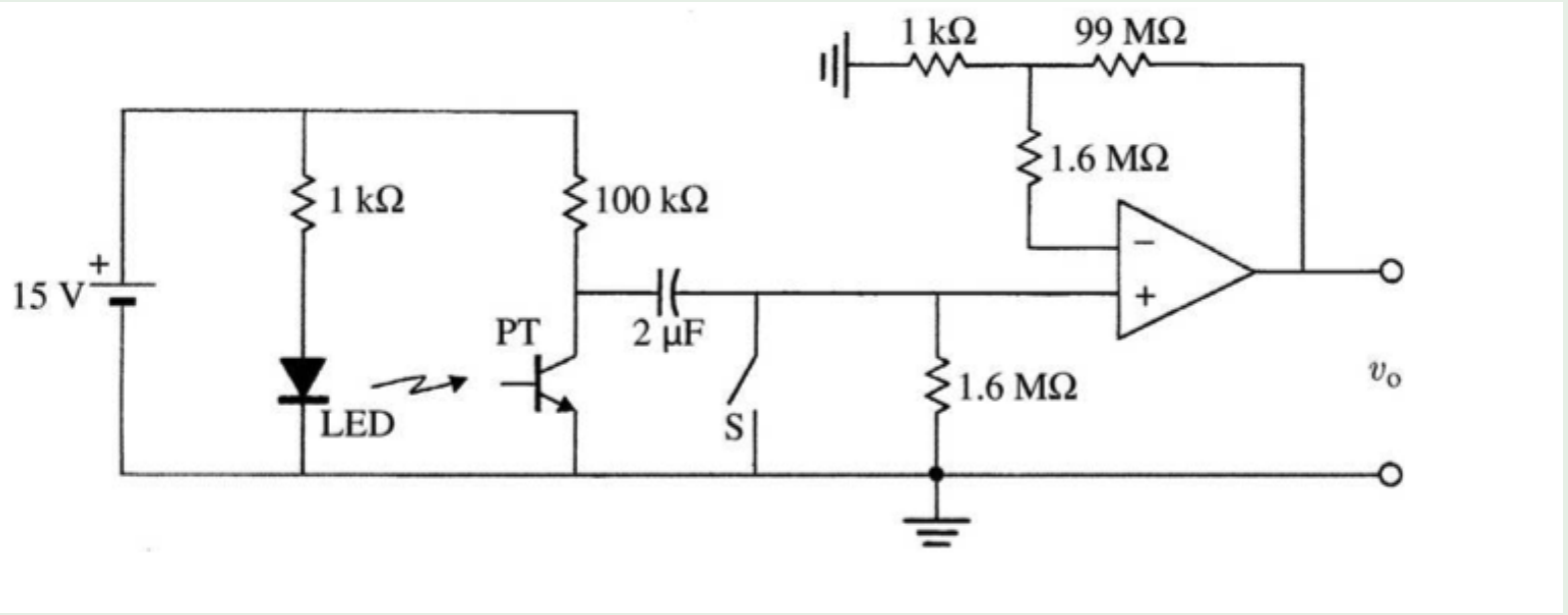


Light transmitted into the finger pad is reflected off bone and detected by a photosensor.

Light transmitted through the aural pinna is detected by a photosensor.

Output of LED is altered by tissue absorption to modulate the phototransistor. The DC level is blocked by the capacitor, and switch S restores the trace. Amplifier drives low impedance loads and provides a gain of 100.

Design of Solid-state Plethysmographic Circuit



Typical LED current is 15mA.
$R_L = \frac{15}{1.5 \cdot 10^{-3}} = 1K\Omega$.
Typical PT current is 150 μA.
$R_p = \frac{15}{150 \cdot 10^{-6}} = 100K\Omega$
Largest capacitor $C = 2\mu$F.
Output resistor $R_o = 1/(2\pi f_o C) = 1/(2\pi 0.05 \cdot 2 \cdot 10^{-6}) = 1.6M\Omega$