BLOOD PRESSURE and SOUND MEASUREMENTS

Lecture Notes

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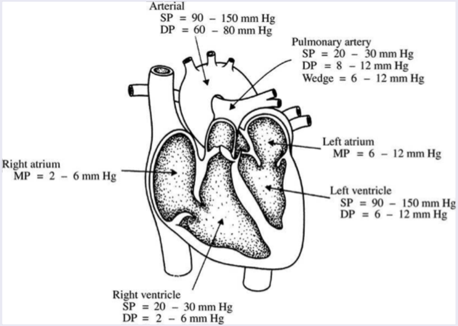
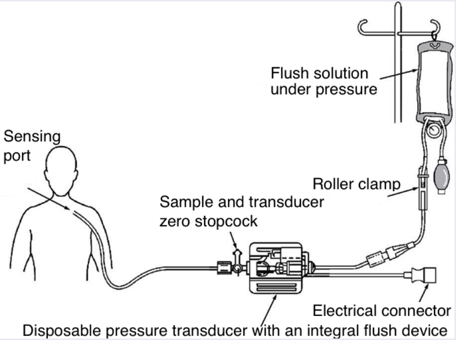
Typical Values of Circulatory Pressures



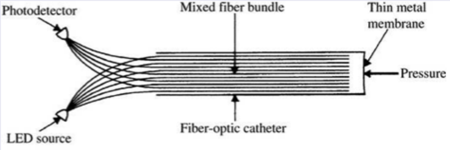
SP : systolic pressure
DP : diastolic pressure
MP : mean pressure

Indirect Measurements

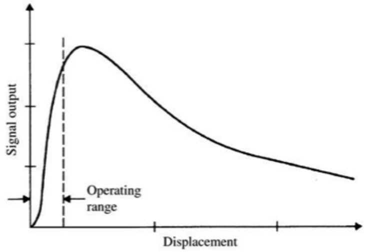


Extravascular pressure sensor system
A catheter couples a flush solution (heparinized saline) through a disposable pressure sensor with an integral flush device to the sensing port.
The three-way stopcock is used to take blood samples and zero the pressure sensor.

Indirect Measurements

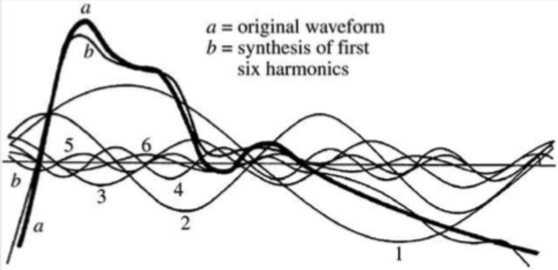


Intravascular fiber-optic pressure sensor
Pressure causes deflection in a thin metal membrane that modulates the coupling between the source and detector fibers.



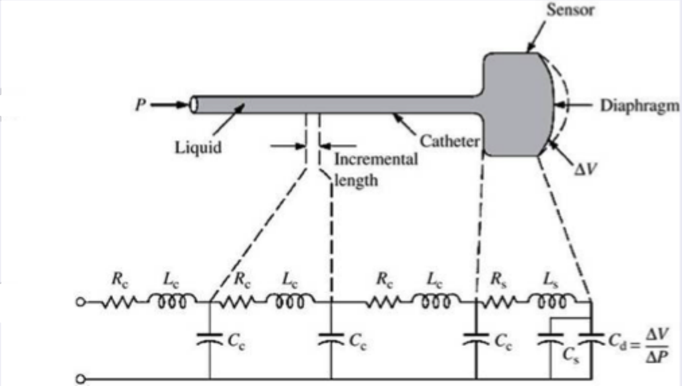
Characteristic curve for the fiber-optic pressure sensor

Blood pressure waveform



Harmonic	Amplitude (%)
1	100
2	63.2
3	29.6
4	22.2
5	14.8
6	11.8

Physical Model of a Catheter-sensor System



$R_c = \frac{\Delta P}{F}(Pa \cdot s/m^3)$ or $R_c = \frac{\Delta P}{\bar{u}A}$

P : Pressure difference

V : Volume

F : Flow rate

$\bar{u}$: Average velocity

A : cross-section area

$L_c = \rho \frac{L}{\pi r^2}$: Inertance of the fluid

Poiseuille's equation

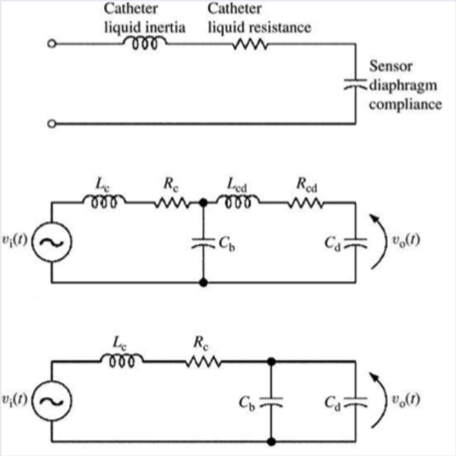
$R_c = \frac{8\eta L}{\pi r^4}(Pa \cdot s/m^3)$: Resistance of the fluid

$C_c = \frac{\Delta V}{\Delta P}$: Compliance of the fluid

ρ density and η viscosity of the fluid

L Length and r cylindrical radius of the tube

Simplified Analogous Circuits



C_d is larger than C_c or the compliance of sensor cavity for a bubble-free noncompliant catheter. R_c and L_c are larger than those of the sensor, because the catheter has longer length and smaller diameter.

Catheter sensor system with a bubble in the catheter. Catheter properties proximal to the bubble are inertance L_c and resistance R_c, those distal are L_{cd} and R_{cd}. Bubble compliance is C_b. *Catheter sensor system with a bubble in the catheter*, assuming that L_{cd} and R_{cd} are negligible compared to R_c and L_c.

Input-output Relationship of Transducer Catheter System

Liquid inertance of the catheter due the liquid mass

$L_c = \frac{\Delta P}{dF/dt} = \frac{ma/A}{dA \cdot v/dt} = \frac{ma/A}{Aa} = \frac{m}{A^2} = \frac{\rho L}{\pi r^2}(Pa \cdot s^2/m^3)$

Compliance of diaphram

$C_d = \frac{\Delta V}{\Delta P} = \frac{1}{E_d}$ E_d: Young Modulus

$$v_i(t) = L_c C_d \frac{d^2 v_o(t)}{dt^2} + R_c C_d \frac{dv_o(t)}{dt} + v_o(t)$$

Natural frequency $\omega_n = 1/\sqrt{L_c C_d}$

Damping ratio $\xi = \frac{R_c}{2}\sqrt{\frac{C_d}{L_c}}$

Natural frequency in Hz $f_n = \frac{r}{2}\left(\frac{1}{\pi \rho L}\frac{\Delta P}{\Delta V}\right)^{1/2}$

Damping ratio $\xi = \frac{4\eta}{r^3}\left(\frac{L}{\pi \rho}\frac{\Delta V}{\Delta P}\right)^{1/2}$

Air Bubble in a Catheter

A 5 mm-long air bubble has formed in the rigid walled catheter connected to a Statham P23Dd sensor. The catheter is 1 m long, 6 French diameter, and filled with water at 20^oC. The isothermal compression of air $\Delta V/\Delta P$ is 1 ml/cm of water pressure per liter of volume. Plot the frequency-response curve of the system with and without the bubble. Internal radius of the catheter is 0.46 mm, volume modulus of elasticity of the diaphragm (E_d) is 0.49 $\cdot 10^{15}$ $\frac{N}{m}$ and water viscosity $\eta = 10^{-3} Pa \cdot s$

f_n and ξ_n without bubble

$$f_n = \frac{r}{2}\left(\frac{1}{\pi\rho L}\frac{\Delta P}{\Delta V}\right)^{1/2} = \frac{0.46\cdot 10^{-3}}{2}\left(\frac{1}{\pi\cdot 10^3}\frac{0.49\cdot 10^{15}}{1}\right)^{1/2} = 91 \text{ Hz}$$

$$\xi = \frac{4\eta}{r^3}\left(\frac{L}{\pi\rho}\frac{\Delta V}{\Delta P}\right)^{1/2} = \frac{4\cdot 10^{-3}}{(0.46\cdot 10^{-3})^3}\left(\frac{1}{\pi}\frac{1}{10^3\cdot 0.49\cdot 10^{15}}\right)^{1/2} = 0.033$$

f_n and ξ_n with bubble

$$C_d = \frac{\Delta V}{\Delta P_d} = \frac{1}{E_d} = 2.04\cdot 10^{-15}$$

Bubble volume in m^3:

$$\pi r^2 l = \pi(0.46\cdot 10^{-3})^2\cdot 5\cdot 10^{-3} = 3.33\cdot 10^{-9} m^3 = 3.33\cdot 10^{-6} lt$$

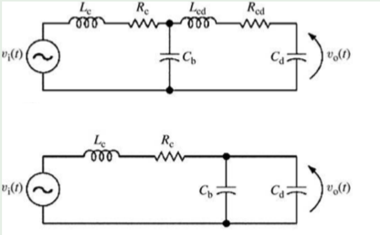
1 cm water pressure per liter volume of air in $N\,(kg\,m/s^2)$:

$$\frac{1g/cm^3\cdot 1cm^3}{1cm^2}\cdot 9.8m/s^2 = \frac{10^{-3}kg}{10^{-4}m^2}\cdot 9.8m/s^2 = 98\frac{N}{m^2}$$

Compliance per liter volume of air : $\frac{\Delta V}{\Delta P} = \frac{10^{-3}lt}{98\,N/m^2} \,/\, lt$

For a volume of $3.33\cdot 10^{-6}lt$, the compliance is

$$C_b = 3.33\cdot 10^{-6} lt\frac{10^{-3}lt}{98N/m^2} \,/\, lt = 3.4\cdot 10^{-14}\frac{m^5}{N}$$



f_n and ξ_n with bubble $f_{n,b} = 91 \left(\frac{2.04 \cdot 10^{-15}}{3.38 \cdot 10^{-14}}\right)^{1/2} = 22$ Hz

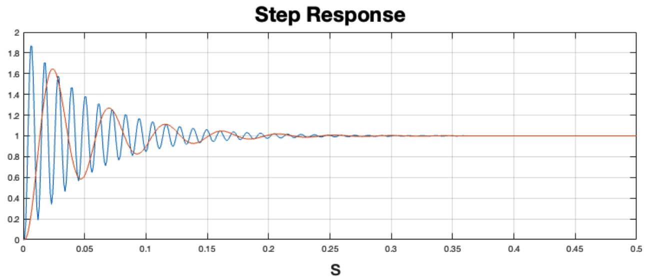
$$\xi_b = \xi \left(\frac{(\Delta V/\Delta P)_t}{\Delta V/\Delta P}\right)^{1/2} = 0.138$$

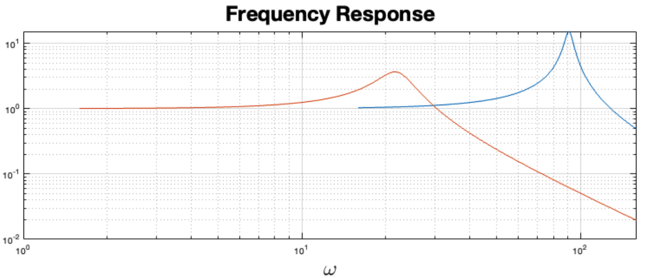
$$C_t = C_d + C_b = 2.04 \cdot 10^{-15} + 3.4 \cdot 10^{-14} \sim 3.58 \cdot 10^{-14}$$

Frequency Response of A Catheter with Bubble

```
% Step response of 2nd order system natural frequency
% and Damp coefficient
xi=[0.033 0.138];
wn=[91 22]*2*pi;
t=0:0.001:.5;
for K=1:2
  num = wn(K)^2;
  denom = [1 2*xi(K)*wn(K) wn(K)^2];
  Y(:,K)=lsim(num,denom,sign(t),t);
end;
subplot(2,1,1);plot(t,Y);grid

% MATLAB SCRIPT FOR  frequency %response
xi=[0.033 0.138];
wn=[91 22]*2*pi;
for K=1:2
  num = wn(K)^2;
  denom = [1 2*xi(K)*wn(K) wn(K)^2];
  [H(:,K),W(:,K)] = freqs(num,denom);
end; subplot(2,1,2);
loglog(W/(2*pi),abs(H));grid;
```


Step Response
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
S


Frequency Response
10^1
10^0
10^{-1}
10^{-2}
10^0
10^1
10^2
ω

Self Study Questions

Water is not a perfect liquid because it has a finite volume modulus of elasticity. Therefore, a theoretical upper limit of high frequency response exists for a water filled sensor catheter system. Find the maximal f_n for a sensor that has a 0.5 ml liquid chamber and that is connected to the pressure source by means of a #20 (r=0.29 mm) 50 mm long steel needle.

A steel needle for the catheter $\rightarrow E_p = 0$

$1ml = 10^{-3} \cdot 10^{-3} m^3 = 10^{-6} m^3 = (10^{-2} m)^3 = 1cm^3$

$\Delta V / \Delta P$ for water : $0.53 \cdot 10^{-15} \frac{m^5}{N}$ per ml volume

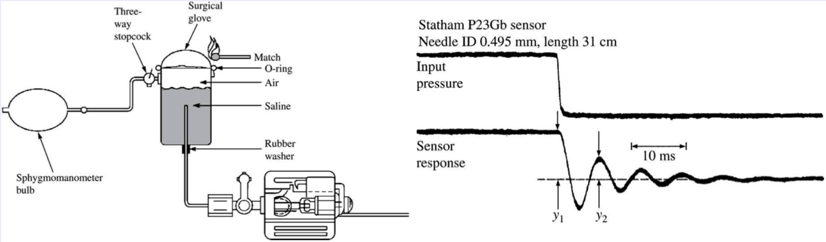
Total volume of water in ml:

$V_t = 0.5ml + \pi(0.029cm)^2 \cdot 5cm = 0.513cm^3 = 0.513ml$

$C_w = \Delta V / \Delta P = 0.53 \cdot 10^{-15} \frac{m^5}{N \cdot ml} \cdot 0.513ml = 0.272 \cdot 10^{-15} \frac{m^5}{N}$

$f_n = \frac{r}{2} \left(\frac{1}{\pi \rho L} \frac{\Delta P}{\Delta V} \right)^{1/2} = \frac{0.029 \cdot 10^{-2}}{2} \left(\frac{1}{\pi \cdot 10^3 \cdot 0.05 \cdot 0.27 \cdot 10^{-15}} \right)^{1/2} = 700 \text{ Hz}$

Pressure sensor transient response.

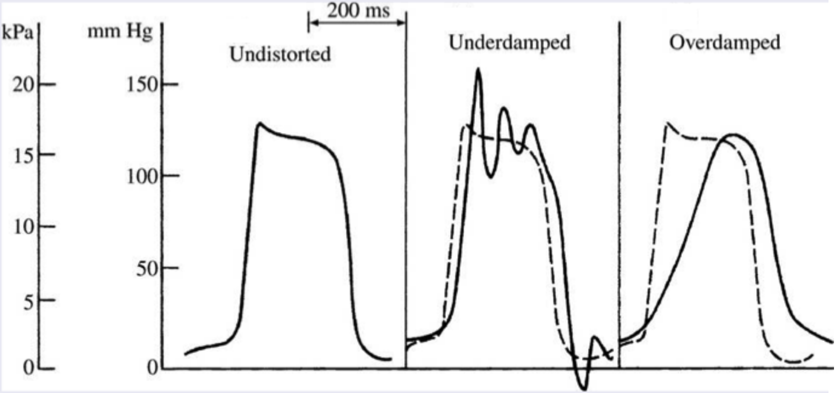


Top Channel : Negative-step input pressure is recorded on the top channel.
Bottom channel : Sensor response for a Statham P23Gb sensor connected to a 31 cm needle (0.495 mm ID).
For an underdamped response
$y(t) = -\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\sqrt{(1-\xi^2)}\omega_n t + \phi) + K,\ \phi = \arcsin(\sqrt{1-\xi^2})$
Maxima occur at $\sqrt{(1-\xi^2)}\omega_n t + \phi = \frac{3\pi}{2},\ \sqrt{(1-\xi^2)}\omega_n t + \phi = \frac{7\pi}{2}$ etc.
$t_1 = \frac{\frac{3\pi}{2} - \phi}{\omega_n\sqrt{1-\xi^2}},\ t_2 = \frac{\frac{7\pi}{2} - \phi}{\omega_n\sqrt{1-\xi^2}}$
Ratio of the first and second overshoots.
$\frac{y(t_1)}{y(t_2)} = \ln\left(\frac{2\pi\xi}{\sqrt{1-\xi^2}}\right)$

Bandwidth Requirements for Measuring Blood Pressure

- Bandwidth requirements for a heart rate of 120bpm (2 Hz) would be 20 Hz.
- Measurement of the derivative of the pressure of the signal increase the bandwidth requirements, because the derivative of a sinusoid increases the amplitude of that component by a factor proportional to its frequency.
- The amplitude vs frequency characteristics of any catheter manometer system used for the measurement of ventricular pressures that are subsequently differentiated must be flat to within 5% up to the 20th harmonic.

Pressure-waveform distortion

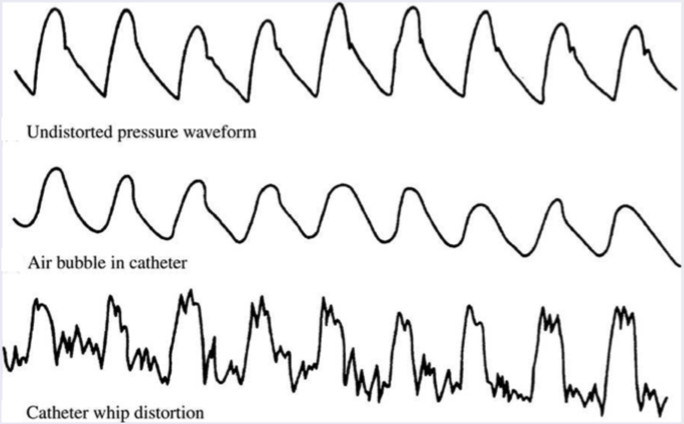


Recording of an undistorted left-ventricular pressure waveform via a pressure sensor with bandwidth dc to 100 Hz.

Underdamped response, where peak value is increased.
A time delay is also evident in this recording.

Overdamped response that shows a significant time delay and an attenuated amplitude response.

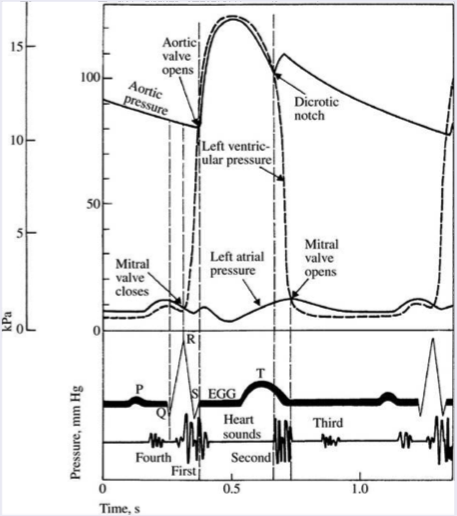
Distortion During the Recording of Arterial Pressure



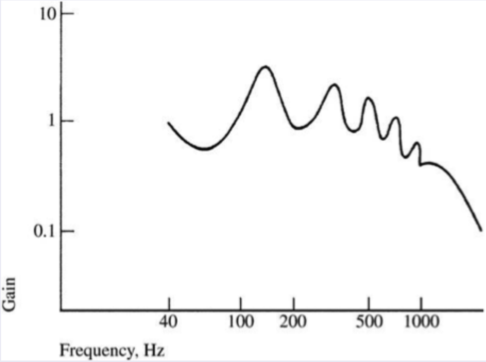
The bottom trace is the response when the pressure catheter is bent and whipped by accelerating blood in regions of high pulsatile flow.

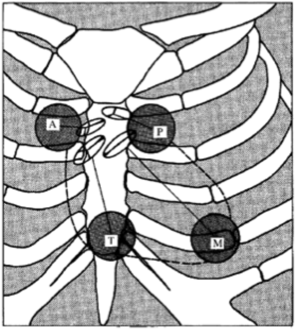
Phonocardiography: Correlation of the heart sounds with electric and mechanical events of the cardiac cycle.

First heart sound
Movement of blood due to ventricular stroke.
Second heart sound
Vibration due to deceleration of flow in aorta and pulmonary artery with closure of semilunar valves.
Third heart sound
due to suddent termination of rapid filling phase of ventricles from atria.
Fourth heart sound
occurs when atria contract and propel blood into ventricles.



Auscultation





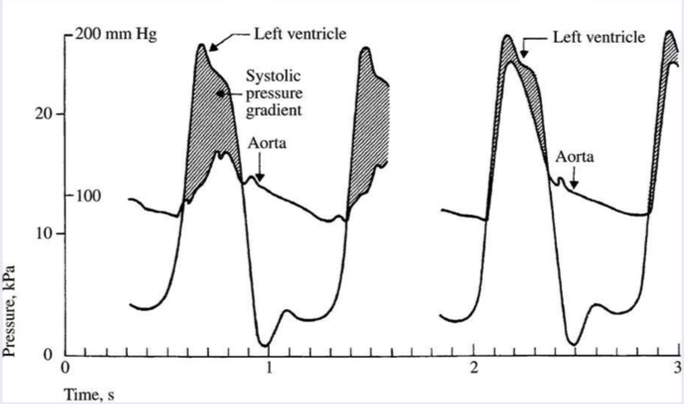
Frequency Response of stethescope
Frequency Range : 0.5 to 2 KHz

Auscultatory areas on the chest
A : aortic
P : pulmonary
T : tricuspid
M : mitral.

Phonocardiography : Recoring heart sounds and murmurs.

A microphone system with a frequency response from 0.1-100 Hz.
The cardiologists evaluates the results of a phonocardiograph on the basis of change changes in wave shape and in a number of timing parameters.

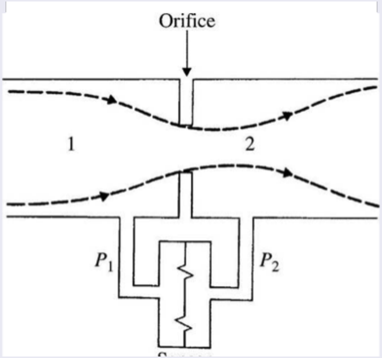
Aortic Stenosis Measured by Cardiac Catheterization



Systolic pressure gradient (left ventricular-aortic pressure) across a stenotic aortic valve.

Marked decrease in systolic pressure gradient with insertion of an aortic ball valve.

Model for Deriving Equation for Heart valve Orifice Area

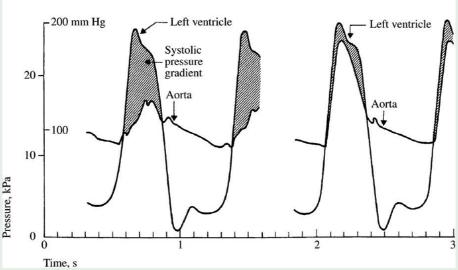


$$P_t = P + \rho g h + \rho u^2/2$$
$$P_1 - P_2 = \frac{\rho u^2}{2}$$
$$A = \frac{F}{u} = F\left(\frac{\rho}{2(P_1 - P_2)}\right)^{1/2}$$
$$A = \frac{F}{c_d}\left(\frac{\rho}{2(P_1 - P_2)}\right)^{1/2}$$

c_d : discharge coefficient due to friction

P_1 and P_2 are upstream and downstream static pressures.
Velocity u is calculated for minimal flow area A at location 2.

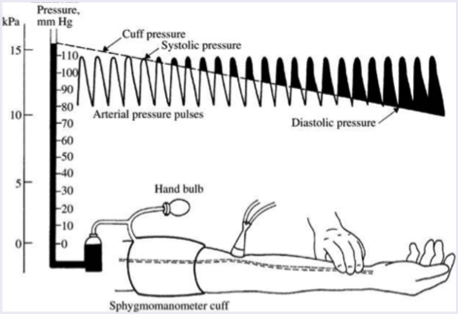
Calculate the approximate area of the aortic valve for the patient with aortic and left ventricular pressures as shown in the figure. Patient's cardiac output is 6400 ml/min and the heart rate as 78 bpm. $\rho_{blood} = 1060$ kg/m^3.



Ejection period : 0.31 s
Average pressure drop : 7.33 KPa

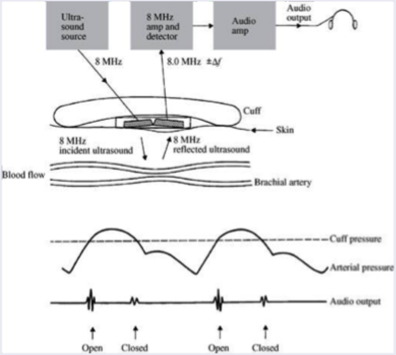
$6400 \cdot 10^{-6} m^3$ blood in 1 min or in 78 beats $\longrightarrow \frac{6.4 \cdot 10^{-3}}{78} m^3$ blood in one beat
Ejection occurs in 0.31 s of each beat
$\frac{6.4 \cdot 10^{-3}}{78} m^3$ ejection occurs in 0.31s $\longrightarrow \frac{6.4 \cdot 10^{-3}}{78 \cdot 0.31} m^3$ ejection occurs per second
Therefore, flow is
$F = 6400 \cdot 10^{-6} m^3 \frac{1}{78} \frac{1}{0.31} = 264 \cdot 10^{-6} m^3/s$
and area is
$A = \frac{264 \cdot 10^{-6}}{0.85} \left(\frac{1060}{2 \cdot 7330}\right)^{1/2} = 83 \cdot 10^{-6} m^2 = 83 mm^2$

Indirect Blood Pressure Measurement. The sphygmomanometer.



The cuff is inflated by a hand bulb to pressure above the systolic level.
Pressure is then slowly released and blood flow under the cuff is monitored by a microphone or stethoscope placed over a downstream artery.
The first Korotkoff sound detected indicated systolic pressure, whereas the transition from muffling to silence brackets diastolic pressure.

Indirect Blood Pressure Measurement. Ultrasonic Method.



Ultrasonic determination of blood pressure

A compression cuff is placed over the transmitting (8 MHz) and receiving (8 MHz $\pm\Delta$f) crystals.

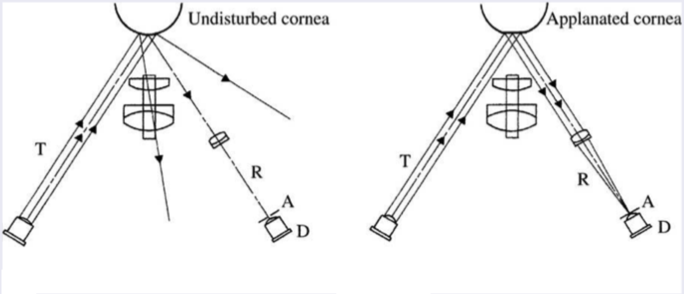
The opening and closing of the blood vessel are detected as the applied cuff pressure is varied.

Δf is between 40 to 500 Hz proportional to the velocity of the wall motion and the blood velocity.

When cuff presure is above systolic pressure no doppler signal is observed. When the pressure reduces to the level at which the vessel starts to open, a signal is detected indicating the systolic level.

As the cuff pressure reduces to the diastolic level, the artery becomes no more occluded and the pulse signal due to doppler shift becomes very small indicating the diastolic level.

Intraocular Pressure Measurement with Tonometry



An air pulse of linearly increasing forms deforms and flattens the central cornea arae within miliseconds.
If the cornea is applanated, it acts like a plane mirror with a resulting maximum signal detection.
There is linear relation between intraocular pressure and time interval to applanation.