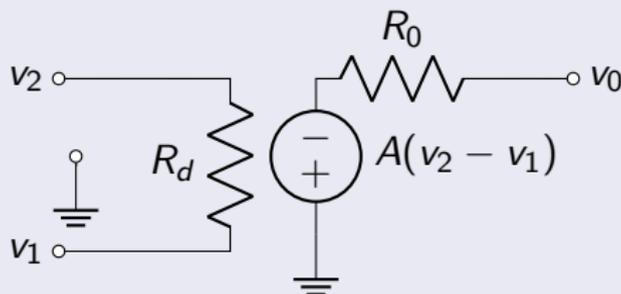


AMPLIFIERS

Lecture Notes

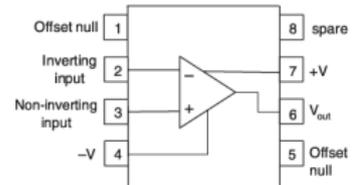
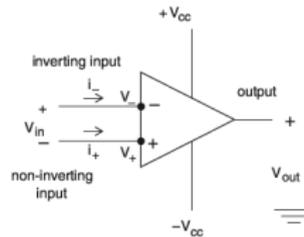
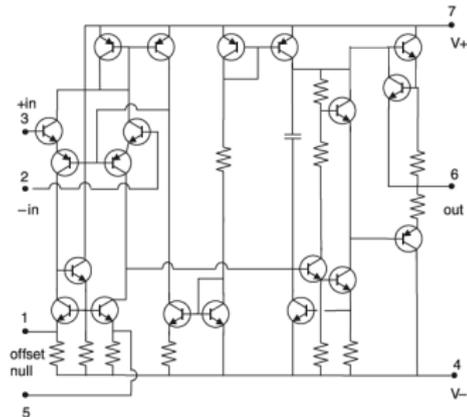
Ahmet Ademoglu, *PhD*
Bogazici University
Institute of Biomedical Engineering

IDEAL OP-AMP

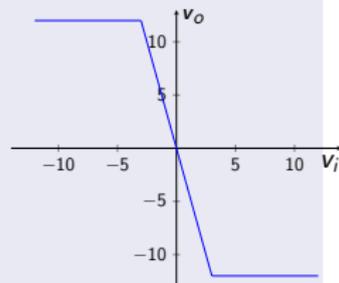
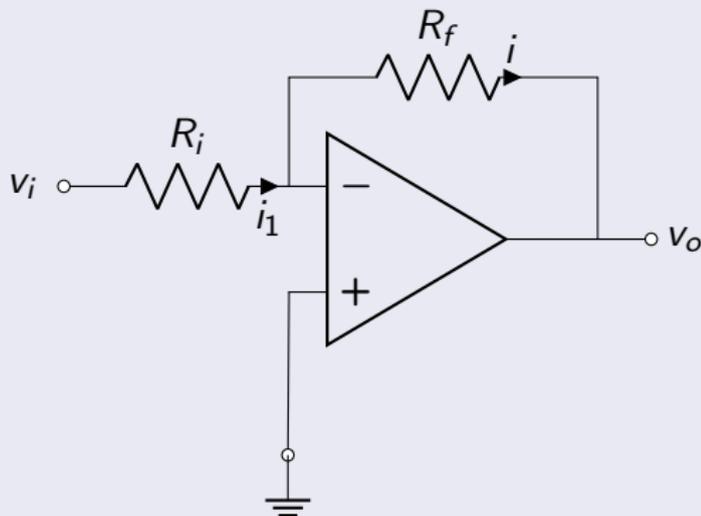


- $A = \infty$ (gain is infinite)
- $v_0 = 0$, when $v_1 = v_2$ (no offset voltage)
- $R_d = \infty$ (input impedance is infinite)
- $R_0 = 0$, (output impedance is zero)
- Bandwidth = ∞
(no frequency response limitations, no phase shift)

Typical transistor layout of an op-amp.

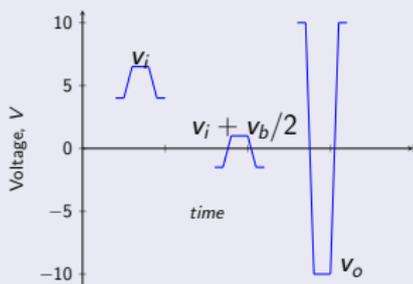
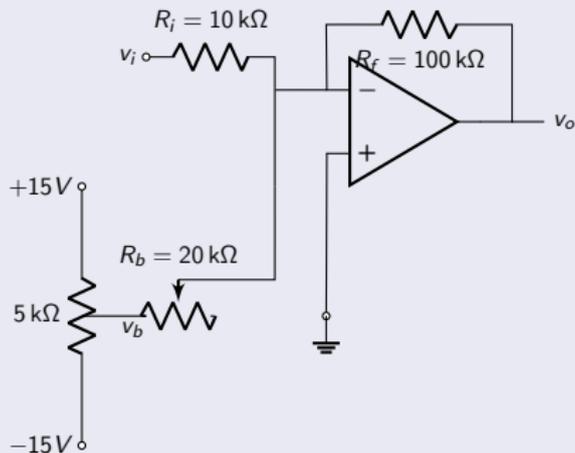


Inverting Op-Amp



$$v_o = -iR_f = -v_i \frac{R_f}{R_i}, \text{ or } \frac{v_o}{v_i} = -\frac{R_f}{R_i}$$

Summing Amplifier

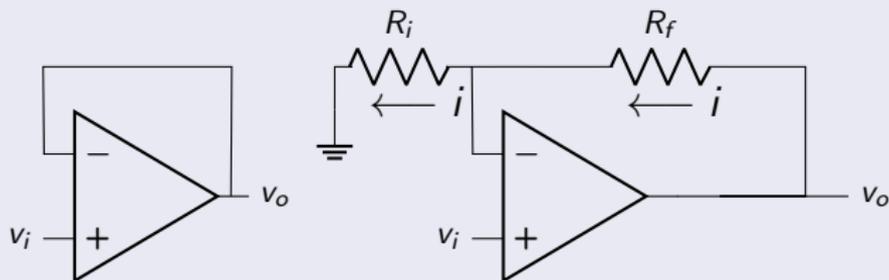


The output of a biopotential preamplifier that measures EOG is an undesired DC voltage of $\pm 5V$ with a desired signal of $\pm 1V$ superimposed. Design a circuit that would balance the DC voltage to 0 and provide a gain of 10 for the designed signal without saturating the op amp.

$$\frac{v_i}{R_i} + \frac{v_b}{R_b} = 0 \longrightarrow R_b = -\frac{R_i v_b}{v_i} = -\frac{10^4(-10)}{5} = 2 \times 10^4 \Omega$$

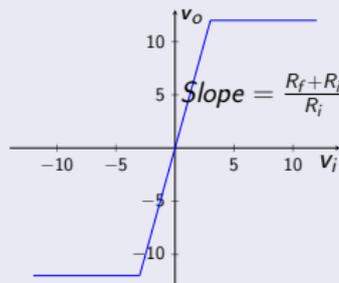
$$\begin{aligned} v_o &= -R_f \left(\frac{v_i}{R_i} + \frac{v_b}{R_b} \right) \\ &= -10^5 \left(\frac{v_i}{10^4} + \frac{v_b}{2 \cdot 10^4} \right) \\ &= -10 \left(v_i + \frac{v_b}{2} \right) \end{aligned}$$

Non-Inverting Amplifier

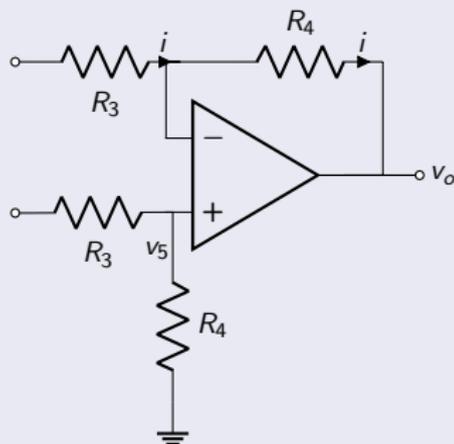
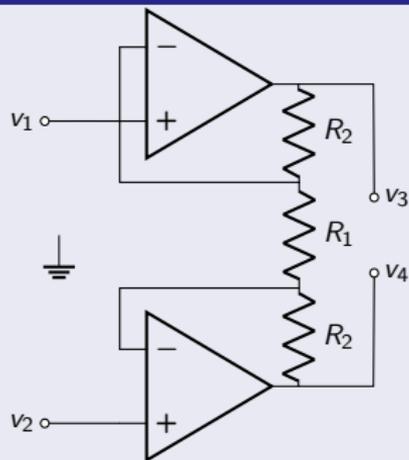


Buffer Circuit : Prevents a high source resistance from being loaded down by a low-resistance load.

$$\frac{v_o}{v_i} = \frac{i(R_i + R_f)}{iR_i} = \frac{R_f + R_i}{R_i}$$



Differential Amplifiers



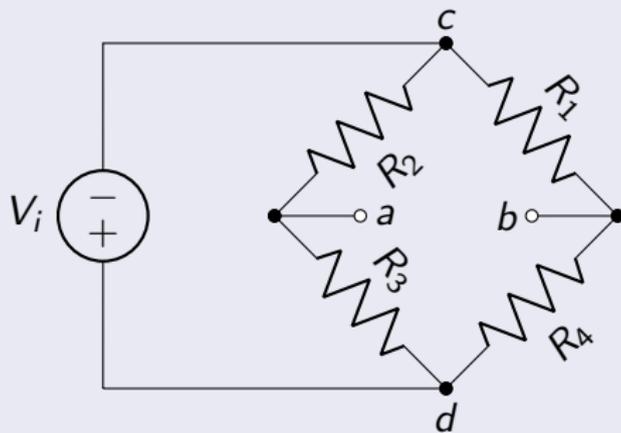
$$v_5 = v_4 \frac{R_4}{R_3 + R_4} \quad i = \frac{v_3 - v_5}{R_3} = \frac{v_5 - v_o}{R_4} \rightarrow v_o = (v_4 - v_3) \frac{R_4}{R_3}$$

Common Mode : $v_3 = v_4$, \rightarrow Gain $G_c = 0$.

Differential Mode : $v_3 \neq v_4 \rightarrow$ Gain $G_d = \frac{R_4}{R_3}$.

$CMRR = \frac{G_d}{G_c}$ must be $> 10^4$ for good quality amplifiers.

Wheatstone Bridge



$$R_{eq} = R_2 \parallel R_3 + R_1 \parallel R_4$$

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} + \frac{R_1 R_4}{R_1 + R_4}$$

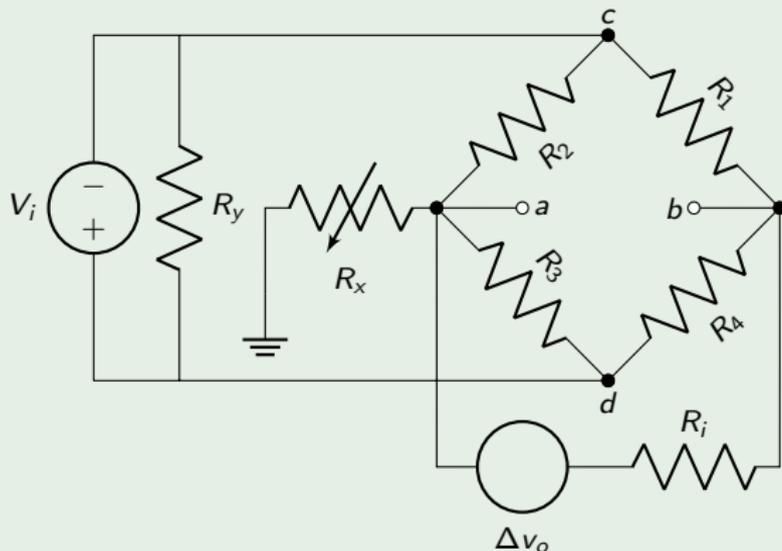
$$V_{eq} = V_{ab} = V_a - V_b$$

$$V_{eq} = V_i \frac{R_3}{R_2 + R_3} - V_i \frac{R_4}{R_1 + R_4}$$

$$V_{eq} = V_i \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_4)(R_2 + R_3)}$$

If R_1 and R_3 increase by ΔR and R_2 and R_4 decrease by ΔR then $V_{ab} = V_i \frac{\Delta R}{R}$ assuming that initially $R_1 = R_2 = R_3 = R_4 = R$.

A blood pressure sensor uses a four active arm Wheatstone strain gage bridge excited with DC. At full scale, each arm changes resistance by $\pm 0.3\%$. Design an amplifier that will provide a full-scale output over the op amps's full range of linear operation. Use the minimal number of components.



If R_1 and R_3 increase by ΔR and R_2 and R_4 decrease by ΔR then

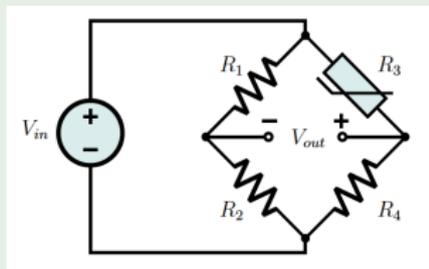
$$\Delta v_o = \frac{\Delta R}{R_0} v_i = \frac{\Delta R}{R} v_i$$

$$\Delta v_o = 5 \cdot 0.003 = 0.015 V.$$

$$Gain = \frac{20}{0.015} = 1333.$$

Assume $R = 120\Omega$, $R_T = 60\Omega$.
Use R_T to replace R_3 and $G_d \cdot R_3 = 60 \cdot 1333 = 80k\Omega$ to R_4 in the amplifier.

Consider the following circuit for a thermometer:



R_3 represents a thermistor with a transfer function $R_3(T) = r_\infty e^{\beta T}$, where β and r_∞ are constants and T is temperature.

- 1 Write the voltage output V_{out} of the system as a function of temperature T . Pay attention to the polarity of V_{out} .

$$V_{out+} = R_4 / (R_3(T) + R_4), \quad V_{out-} = R_2 / (R_1 + R_2), \quad V_{out}(T) = R_4 / (r_\infty e^{\beta T} + R_4) - R_2 / (R_1 + R_2)$$

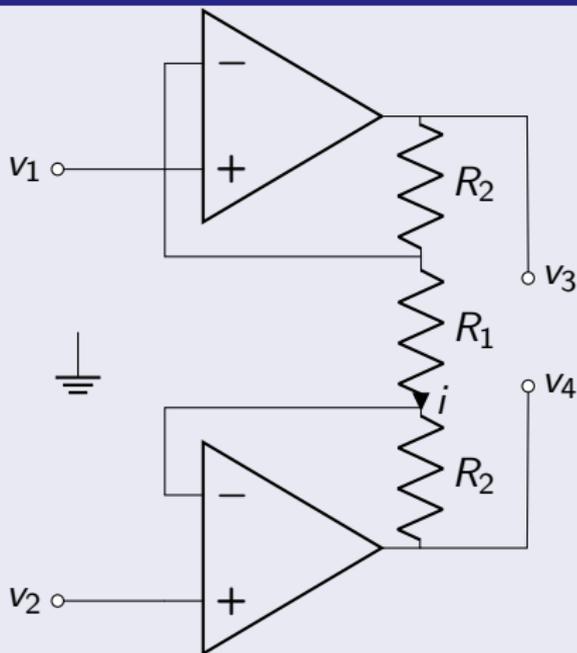
- 2 What is the sensitivity of the thermistor's resistance with respect to temperature? In other words, calculate dR_3/dT .

$$\frac{dR_3}{dT} = r_\infty \beta e^{\beta T}$$

- 3 What is the sensitivity of the voltage output V_{out} with respect to T ?

$$\frac{V_{out}(T)}{dT} = -r_\infty \beta e^{\beta T} R_4 / (r_\infty e^{\beta T} + R_4)^2,$$

3-Op-Amp Differential Amplifier (*Instrumentation Amplifier*)



$$v_3 - v_4 = i(R_2 + R_1 + R_2),$$

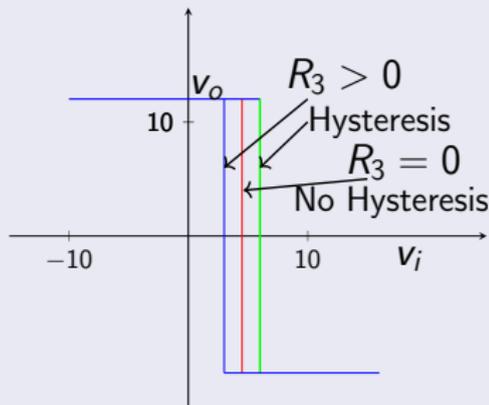
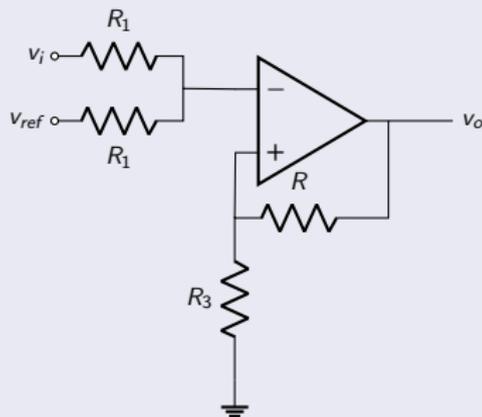
$$G_d = \frac{v_4 - v_3}{v_1 - v_2} = \frac{2R_2 + R_1}{R_1}$$

$$v_1 - v_2 = iR_1$$

$$G_c = 1$$

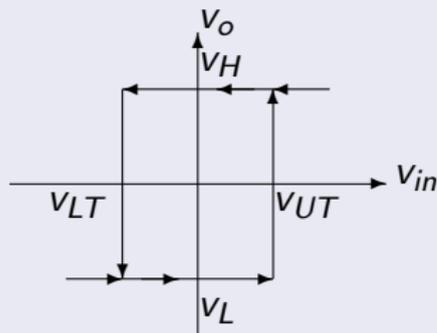
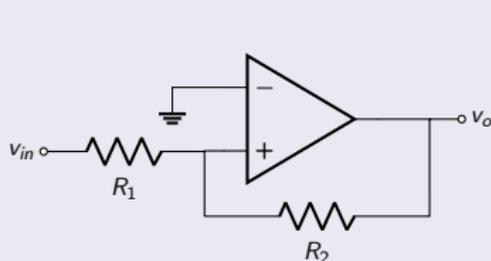


Comparators



Assume that $v_{ref} = -5V$ and $v_i = +10V \rightarrow v_o = -13V$. Let's say the + input is $-1V$, the comparator does not flip until v_i is lowered to $+3V$ which makes the - input equal to the +. At this point, v_o flips to $+13V$ causing the + input to change to $+1V$. The - input must be raised to $+1V$ to cause the next flip v_i to be raised to $+7V$, to have v_o flip back to $-13V$.

Schmitt Trigger



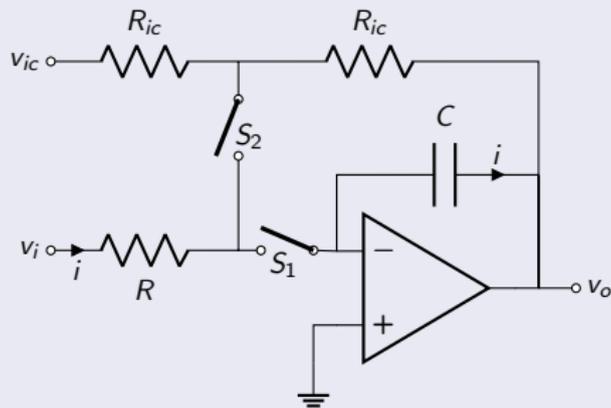
$$\frac{v_{in} - v_{+}}{R_1} = \frac{v_{+} - v_o}{R_2} \rightarrow v_{+} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_{in}}{R_1} + \frac{v_o}{R_2}$$

$$v_{+} = v_{in} \frac{R_2}{R_1 + R_2} + v_o \frac{R_1}{R_1 + R_2}$$

If $v_{+} < 0 \rightarrow v_o = v_L$ If $v_{+} > 0 \rightarrow v_o = v_H$

- Assume $v_o = v_L \rightarrow v_{+} = v_{in} \frac{R_2}{R_1 + R_2} + v_L \frac{R_1}{R_1 + R_2}$
 When v_{+} becomes > 0 then v_o goes from $v_L \rightarrow v_H$ which indicates
 $v_{in} \frac{R_2}{R_1 + R_2} > -v_L \frac{R_1}{R_1 + R_2}$ or $v_{in} > v_{UT} = -v_L \frac{R_1}{R_2}$
- Now assume $v_o = v_H \rightarrow v_{+} = v_{in} \frac{R_2}{R_1 + R_2} + v_H \frac{R_1}{R_1 + R_2}$
 When v_{+} becomes < 0 then v_o goes from $v_H \rightarrow v_L$ which indicates
 $v_{in} \frac{R_2}{R_1 + R_2} < -v_H \frac{R_1}{R_1 + R_2}$ or $v_{in} < v_{LT} = -v_H \frac{R_1}{R_2}$

Integrator



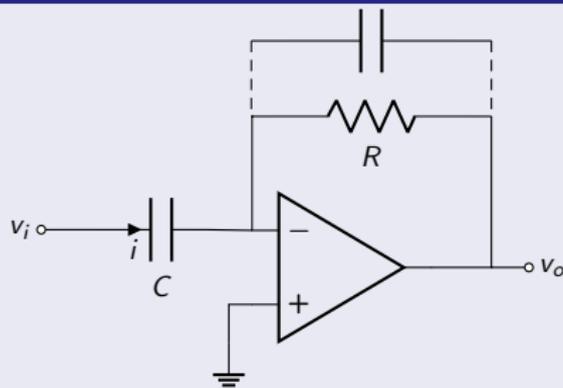
$$v_o = -\frac{1}{RC} \int_0^{t_1} v_i dt + v_{ic}$$
$$\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{Z_f}{Z_i} = -\frac{1}{j\omega\tau}$$

where $\tau = RC$.

A three-mode integrator

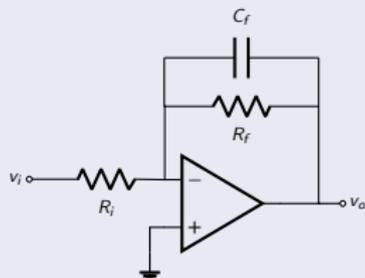
With S_1 open and S_2 closed, the DC circuit behaves as an inverting amplifier. Thus $v_o = v_{ic}$ and v_o can be set to any desired initial condition. With S_1 closed and S_2 open, the circuit integrates. With both switches open, the circuit holds v_o constant, making possible a leisurely readout.

Differentiators

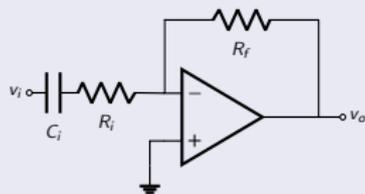


$$\begin{aligned}i &= C \frac{dv_i}{dt} \\v_o &= -RC \frac{dv_i}{dt} \frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} \\&= -\frac{R}{1/j\omega C} = -j\omega\tau\end{aligned}$$

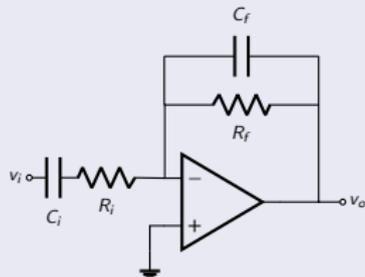
Active Filters



$$\begin{aligned} \frac{V_o(j\omega)}{V_i(j\omega)} &= -\frac{Z_f}{Z_i} = -\frac{R_f/j\omega C_f}{1/j\omega C_f + R_f} \\ &= \frac{R_f}{(1+j\omega R_f C_f)R_i} = -\frac{R_f}{R_i} \frac{1}{1+j\omega\tau} \end{aligned}$$



$$\begin{aligned} \frac{V_o(j\omega)}{V_i(j\omega)} &= -\frac{Z_f}{Z_i} = -\frac{R_f}{1/j\omega C_i + R_i} \\ &= -\frac{j\omega R_f C_i}{1+j\omega R_i C_i} = -\frac{R_f}{R_i} \frac{j\omega\tau}{1+j\omega\tau} \end{aligned}$$

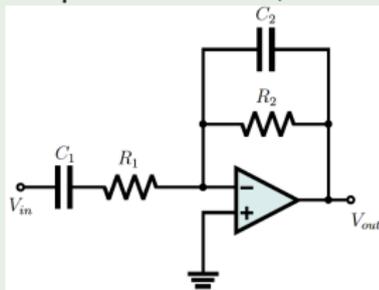


$$\begin{aligned} \frac{V_o(j\omega)}{V_i(j\omega)} &= -\frac{Z_f}{Z_i} = -\frac{R_f/j\omega C_f}{1/j\omega C_i + R_i} \\ &= -\frac{\frac{R_f}{1+j\omega R_f C_f}}{\frac{1+j\omega R_i C_i}{j\omega C_i}} = -\frac{j\omega R_f C_i}{(1+j\omega R_f C_f)(1+j\omega R_i C_i)} \end{aligned}$$



Self Study Question

Consider the following band-pass filter below;



- 1 Find the transfer function $H(j\omega) = V_{out}/V_{in}$. Assume the op-amp is ideal.

$$H(j\omega) = -\frac{j\omega R_2 C_1}{(1+j\omega R_2 C_2)(1+j\omega R_1 C_1)}$$

- 2 Propose values for the circuit components such that the filter has a gain of 30 dB and a pass-band of 20 Hz to 20 kHz.

High pass and lower cut-off $H(j\omega) \approx j\omega R_1 C_1 \rightarrow 2\pi \cdot 20 = 1/R_1 C_1$. Choose $C_1 = 80\mu F$ then $R_1 = 1/(40\pi \cdot 80 \cdot 10^{-6}) = 99.47\Omega$.

Pass band $H(j\omega) \approx -\frac{j\omega R_2 C_1}{\omega R_1 C_1} = 10^{30/20} \rightarrow R_2/R_1 = 31.6$ or $R_2 = 31.6 \cdot 99.47 = 3.143k\Omega$.

Low pass and higher cut-off $H(j\omega) \approx -j\omega R_2 C_2 \rightarrow 2\pi \cdot 20k = 1/(R_2 C_2)$ or $C_2 = 1/(3143 \cdot 2\pi \cdot 2 \cdot 10^4) = 2.5319nF$

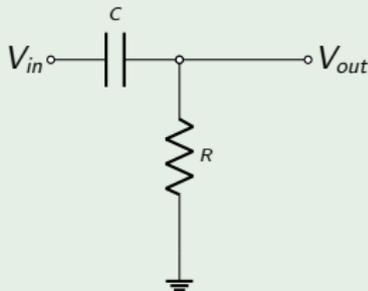
- 3 Draw a Bode plot of the filter response.



Self Study Question

A photoplethysmograph device can be used to determine heart rate. However, it produces voltage signals that are superimposed on top of a large DC component. You need to design a filter to remove the DC component from the voltage signal.

- 1 Draw a generic first order filter circuit suitable for this purpose. Use variables to represent the component values. Label the voltage input and voltage output nodes clearly. What kind of filter is this?



$$\text{High Pass Filter : } \frac{V_{out}}{V_{in}} = \frac{j\omega RC}{1+j\omega RC}$$

- 2 The heart rate of athletes can be as low as 40 beats per minute. Given this information, propose a set of numerical component values that are appropriate for this filter.

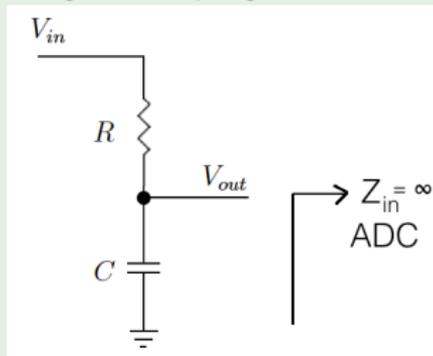
40 beats/min = 2/3 Hz, $f_c = 1/(2\pi RC) < 2/3$. If $f_c = 0.1$ Hz, $C = 1\mu F$, $R = 1/(2\pi f_c C) \approx 1.6M\Omega$

- 3 Draw Bode (magnitude and phase) plots of this filter. Be sure to label the plots, including any significant features and all axes.

Design Problem

Consider an ADC that operates at a sampling rate of 100 kHz to digitize a voltage signal up to 1 V. Assume this ADC has infinite input impedance. Design a first order filter, using only resistors and capacitors to reduce any aliasing effects. Your circuit should not draw more than $10 \mu\text{A}$ of current from the input.

By using the sampling theorem, the cutoff frequency must be $100/2 < 50\text{kHz}$.



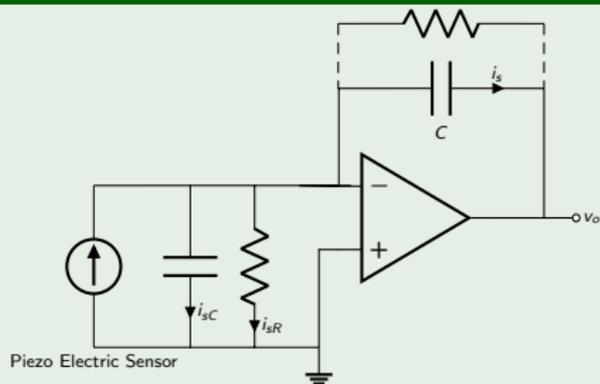
$$2\pi f_c = 1/RC \text{ or } RC = 1/(2\pi \cdot 50 \cdot 10^3)$$

In the worst case $\omega = \infty$ and $1/j\omega C = 0$
which makes $R = \frac{V_{in}}{I_{max}} = \frac{1\text{V}}{10\mu\text{A}} = 100\text{k}\Omega$.

$$C = 1/(2\pi \cdot 50 \cdot 10^3 \cdot 100 \cdot 10^3) = 31.83\text{pF}$$

Manufacturers typically do not produce components of arbitrary values. Instead they produce them using preferred numbers from an E series as component values. C could be 33 pF (33 from the E12 series) instead of 31.83 pF. 33 pF capacitors are commercially available and inexpensive.

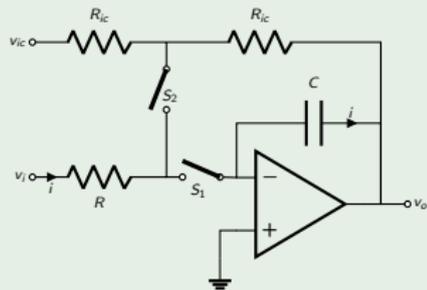
The output of the piezoelectric sensor shown below, may be fed directly into the negative input of the integrator. Analyze the circuit of this charge amplifier and discuss its advantages

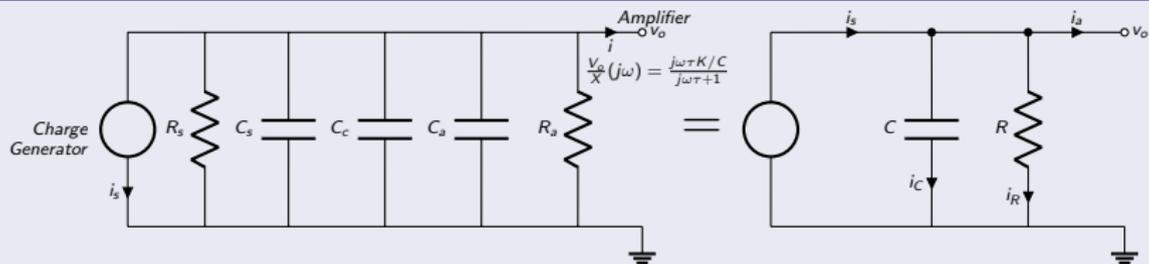


$$i_s = \frac{dq_s}{dt} = K \frac{dx}{dt}$$

$$v_o = -v - \frac{1}{C} \int_0^{t_1} K \frac{dx}{dt} dt = -\frac{KX}{C}$$

The charge amplifier slowly drifts with time because of bias. A large feedback resistance R must therefore be added to prevent saturation. This causes the circuit to behave as a high pass filter, with time constant $\tau = RC$. It then responds only to frequencies above $f_c = \frac{1}{2\pi RC}$.





C_s : Sensor Capacitance, C_c : Cable Capacitance, C_a : Amplifier Capacitance

$$i_s = i_C + i_R$$

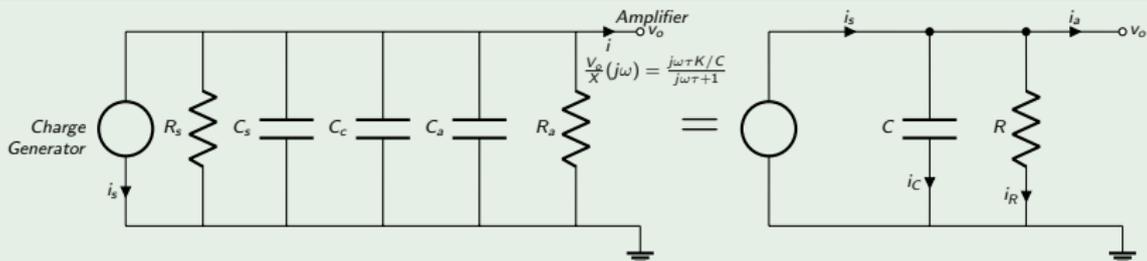
$$v_o = v_C = \frac{1}{C} \int i_C dt$$

$$i_s - i_R = C \left(\frac{dv_o}{dt} \right) = K \frac{dx}{dt} - \frac{v_o}{R}$$

$$\frac{V_o(j\omega)}{X(j\omega)} = \frac{j\omega\tau K/C}{j\omega\tau + 1}$$

$$\tau = RC$$

For a piezoelectric sensor plus cable that has $1nF$ capacitance, design a *voltage amplifier* by using only *one* non-inverting amplifier that has a gain of 10. It should handle a charge of $1\mu C$ generated by the carotid pulse without saturation. It should not drift into saturation because of bias currents. It should have a frequency response from 0.05 to 100 Hz. Add the minimal number of extra components to achieve the design specifications.



C_s : Sensor Capacitance, C_c : Cable Capacitance, C_a : Amplifier Capacitance

$$R = \frac{R_s R_a}{R_s + R_a}$$

$$i_s = K \frac{dx}{dt}$$

$$V = \frac{Q}{C} = \frac{1\mu C}{1nF} = 1kV \text{ too high!}$$

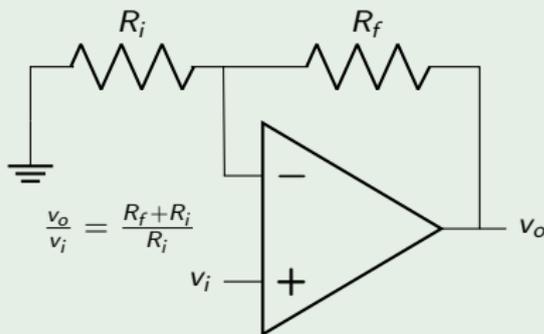
Add a shunt capacitor to bypass C

$C_s = 1\mu F$ to achieve 1V.

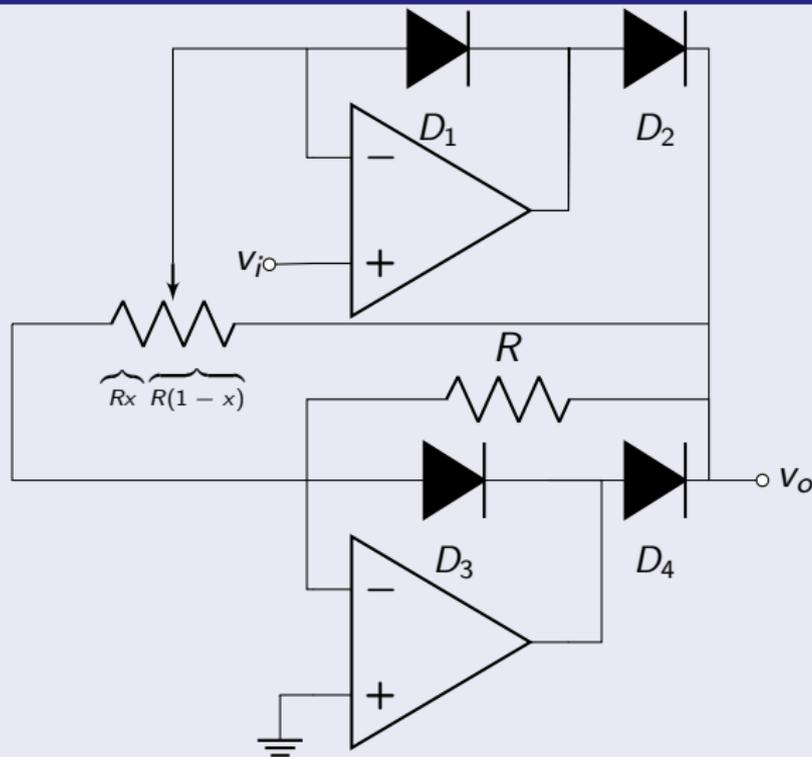
$$R_s = \frac{1}{2\pi f_c C} = \frac{1}{2\pi \cdot 0.05 \cdot 1\mu F} = 3.2M\Omega.$$

$$R_f = 10k\Omega, R_i = 1.1k\Omega$$

$$C_f = \frac{1}{2\pi f_c R_f} = \frac{1}{2\pi \cdot 100 \cdot 10k\Omega} = 160nF.$$



Full Wave Precision Rectifier

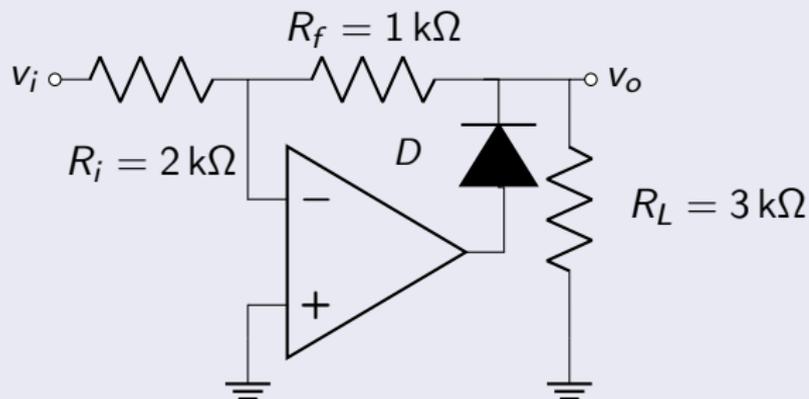


When v_i is (+),
 D_2 and D_3 conduct.
Upper opamp operates
but lower opamp has no
contribution to output.
 $v_o = \frac{v_i}{x}$

When v_i is (-)
 D_1 and D_4 conduct.
Lower opamp operates
but upper opamp has no
contribution to output.
 $v_o = \frac{-v_i}{x}$

Output of the system
is $v_o = \frac{|v_i|}{x}$.

One Opamp Full Wave Rectifier



For (-) v_i , the circuit behaves as an inverting amplifier rectifier with a gain of -0.5 . For (+) v_i , the opamp disconnects and the passive resistor chain yields a gain of 0.5 .

Bode Plots

$$G(s) = \frac{K(s+z_1)(s+z_2)\dots(s+z_k)}{s^m(s+p_1)(s+p_2)\dots(s+p_n)}$$

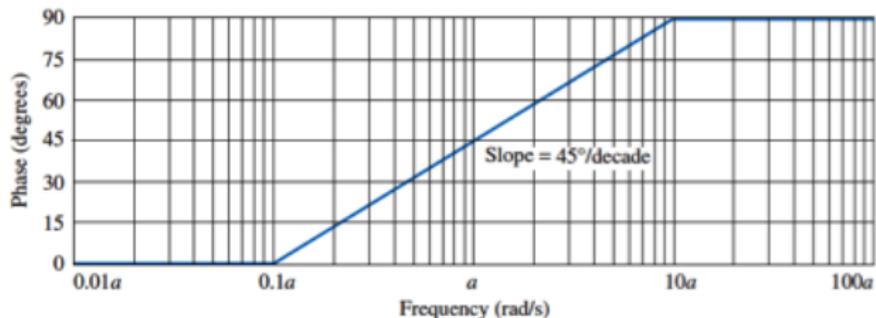
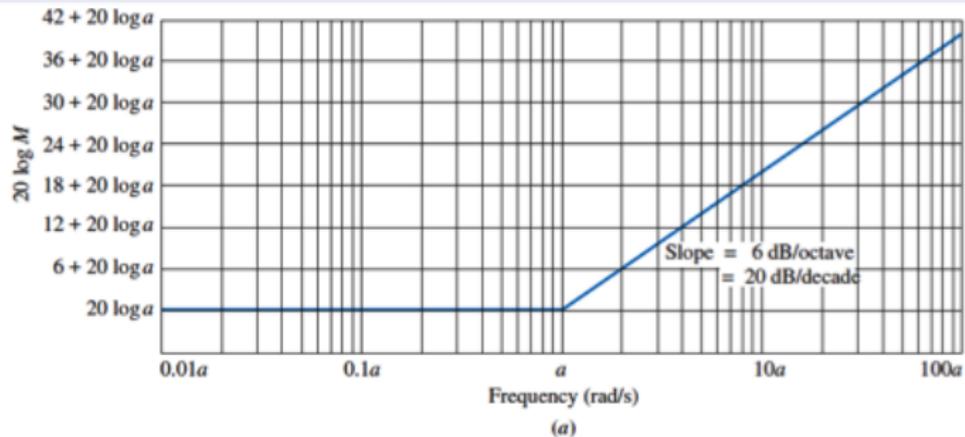
$$20 \log |G(j\omega)|$$

$$= 20 \log(K) + 20 \log |s + z_1| + 20 \log |s + z_2| + \dots + 20 \log |s + z_k| \\ - 20 \log s^m - 20 \log |s + p_1| - \dots - 20 \log |s + p_n|$$

$$G(j\omega) = (j\omega + a) = a \left(j\frac{\omega}{a} + 1 \right)$$

$$G(j\omega) \approx \begin{cases} a & \text{for } \omega \ll a \\ a \left(\frac{j\omega}{a} \right) = \omega e^{j\frac{\pi}{2}} & \text{for } a \ll \omega \end{cases}$$

Bode plot of $G(j\omega) = j\omega + a$.

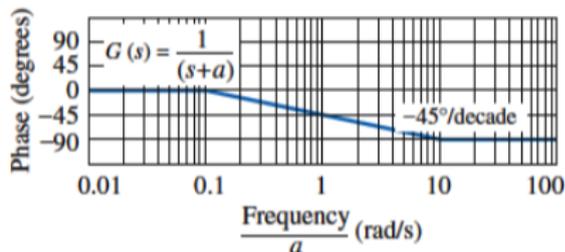
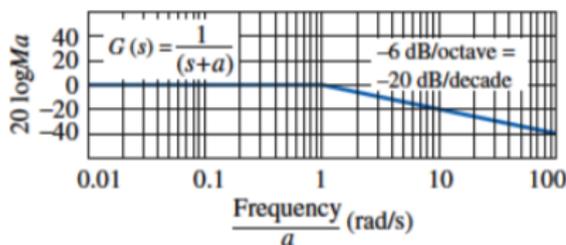


Bode Plots for $G(j\omega) = \frac{1}{j\omega+a}$

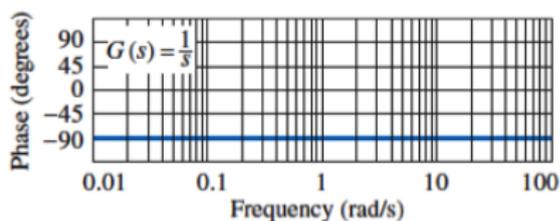
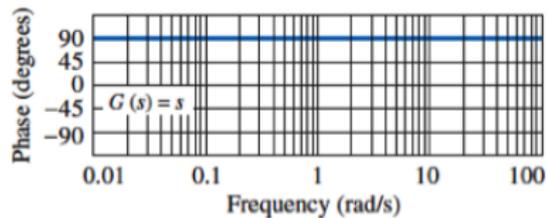
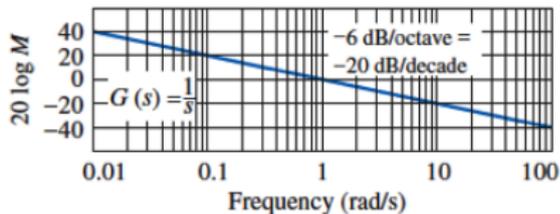
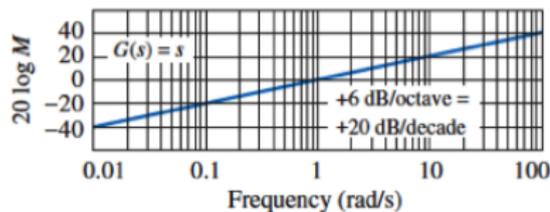
$$G(j\omega) = \frac{1}{j\omega+a} = \frac{1}{a\left(\frac{j\omega}{a}+1\right)}$$

$$G(j\omega) = \frac{1/a}{\left(\frac{j\omega}{a}+1\right)}$$

$$G(j\omega) \approx \begin{cases} \frac{1}{a} & \text{for } \omega \ll a \\ \frac{1/a}{\left(\frac{j\omega}{a}\right)} = \frac{1}{\omega} e^{-j\frac{\pi}{2}} & \text{for } a \ll \omega \end{cases}$$

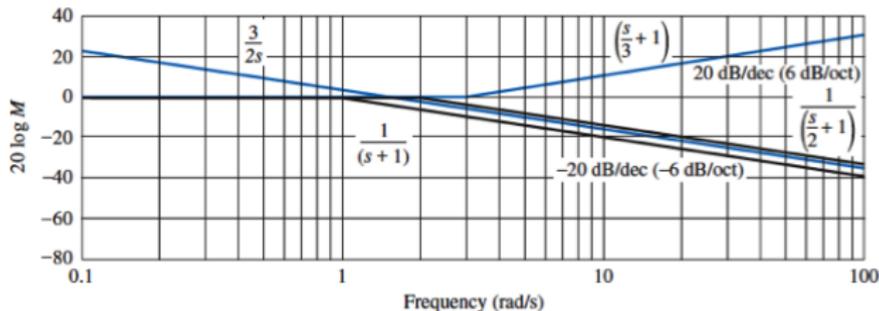


Bode Plots for $G(j\omega) = j\omega$ and $G(j\omega) = \frac{1}{j\omega}$.

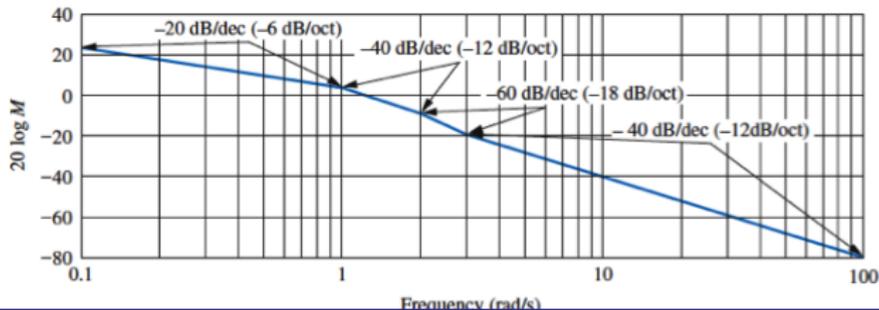


Bode Plots for Ratio of First Order Factors

$$G(j\omega) = K \frac{j\omega+3}{(j\omega)(j\omega+1)(j\omega+2)} = G(j\omega) = \frac{3}{2} K \frac{\left(\frac{j\omega}{3}+1\right)}{j\omega(j\omega+1)\left(\frac{j\omega}{2}+1\right)}$$

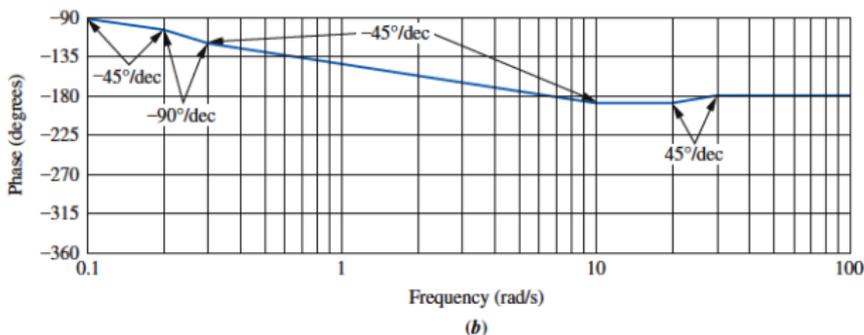
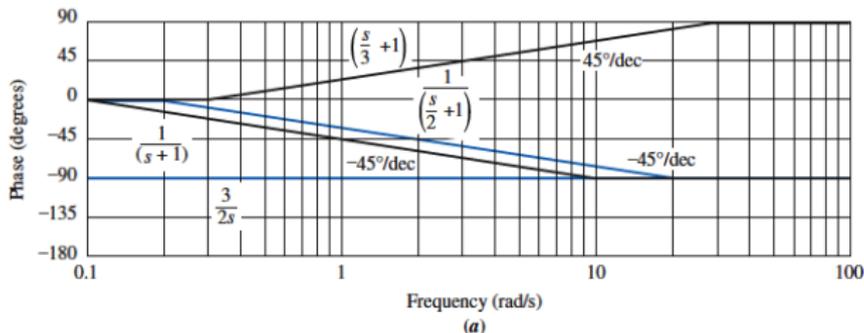


(a)

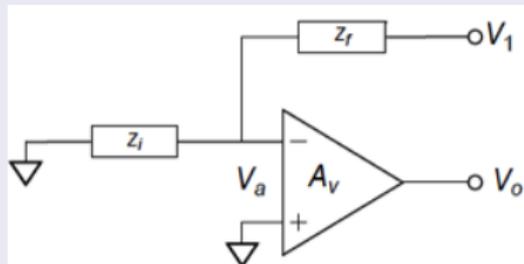
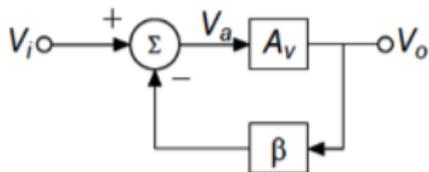
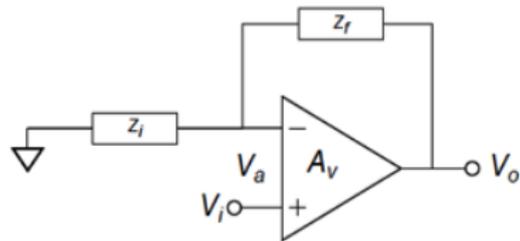


Phase

$$G(j\omega) = \frac{3}{2}K \frac{\left(\frac{j\omega}{3} + 1\right)}{j\omega(j\omega + 1)\left(\frac{j\omega}{2} + 1\right)}$$



Closed Loop Gain of a Non-inverting Amplifier



$$A_{vCL} = \frac{V_o}{V_i} = \frac{A_v}{1 + A_v\beta}$$

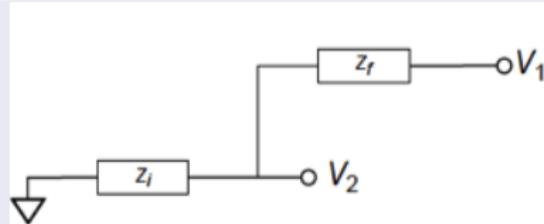
$$= \left(1 + \frac{z_f}{z_i} \left(\frac{1}{1 + \frac{1}{A_v\beta}} \right) \right)$$

Block Diagram

Loop Gain : $A_{VL} = \frac{V_o}{V_1} = -A_v\beta$



Feedback Factor, β



$$\beta = \frac{V_2}{V_1} = \frac{Z_i}{Z_i + Z_f}$$

$$\alpha = 1 - \beta = \frac{Z_f}{Z_i + Z_f}$$

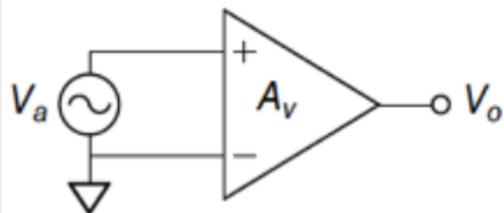
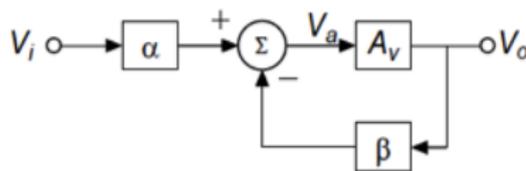
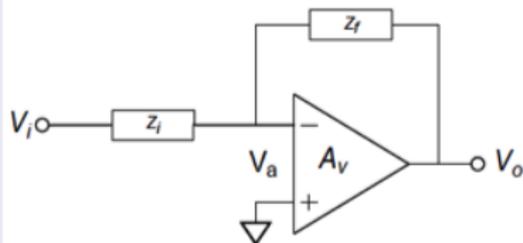
Closed Loop Gain A_{vCL} of an Non-Inverting Amplifier

$$\frac{-V_i + V_a}{Z_i} + \frac{V_o - V_i + V_a}{Z_f} = 0$$

$$-V_i \left(\frac{1}{Z_i} + \frac{1}{Z_f} \right) + V_o \left(\frac{1}{A_v Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_f} \right) = 0$$

$$A_{vCL} = \frac{V_o}{V_i} = \frac{A_v}{1 + A_v \left(\frac{Z_i}{Z_i + Z_f} \right)} = \frac{A_v}{1 + A_v \beta}$$

Closed Loop Gain of an Inverting Amplifier



$$A_{vCL} = \frac{V_o}{V_i} = -\frac{A_v \alpha}{1 + A_v \beta}$$
$$= -\left(\frac{Z_f}{Z_i} \left(\frac{1}{1 + \frac{1}{A_v \beta}} \right) \right)$$

Block Diagram

Loop Gain : $A_{VL} = \frac{V_o}{V_a}$

Closed Loop Gain A_{vCL} for Inverting Amplifier

$$\frac{V_i + V_a}{Z_i} + \frac{V_o + V_a}{Z_f} = 0$$

$$\frac{V_i}{Z_i} + \left(\frac{V_o}{A_v Z_i} + \frac{V_o}{Z_f} + \frac{V_o}{A_v Z_f} \right) = 0$$

$$A_{vCL} = \frac{V_o}{V_i} = -\frac{1}{Z_i} \frac{1}{\frac{1}{A_v Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_f}}$$

$$A_{vCL} = -\frac{A_v \left(\frac{Z_f}{Z_i + Z_f} \right)}{1 + A_v \left(\frac{Z_i}{Z_i + Z_f} \right)} = -\frac{A_v \alpha}{1 + A_v \beta} \quad \text{where } \alpha = \frac{Z_f}{Z_f + Z_i} \text{ and } \beta = \frac{Z_i}{Z_f + Z_i}$$

Calculate the difference between the open and closed loop gains of a noninverting amplifier for a feedback factor i) $\beta = 1$ and ii) $\beta = 0.1$. Assume that amplifier gain $A_v = 100$.

i) The loop gain is $A_v\beta = 100 \cdot 1 = 100$,

$$A_{vCL} = \frac{100}{1+100 \cdot 1} = 0.99$$

Difference between $1/\beta$ and $A_{vCL} = \frac{|1-0.99|}{|1|} = \%1$

ii) The loop gain is $A_v\beta = 100 \cdot 0.1 = 10$,

$$A_{vCL} = \frac{100}{1+100 \cdot 0.1} = 9.09$$

Difference between $1/\beta$ and $A_{vCL} = \frac{|10-9.09|}{|10|} = \%9$

Compensation by a feedback capacitor

If $A_V\beta = -1$ then $A_{VCL} = \infty$

This implies $|A_V\beta| = 1$ and $\angle A_V\beta = \pm 180^\circ$

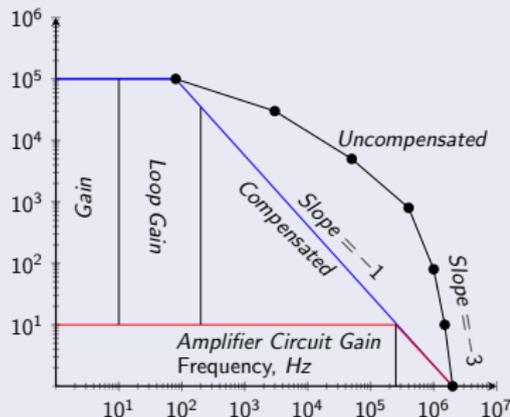
Apply a feedback capacitor C_f in parallel with R_f so that

$$Z_f = 1/sC_f // R_f = \frac{R_f}{1+sC_fR_f}$$

$$1/\beta = \frac{R_i + Z_f}{R_i} = 1 + \frac{R_f/R_i}{j\omega R_f C_f}$$

This is a lowpass filter with phase changing from 0 to -90° which makes the amplifier stabilized.

Frequency Response of Non-Ideal Amplifiers



When feedback is added to the opamp to build an amplifier, the loop gain is the difference between opamp gain and amplifier circuit gain.

If the gain is greater than 1 when the phase shift is -180° , there is undesirable oscillation.

Compensation

Adding an external capacitor, to the feedback compensates the opamp resulting in a slope of -1 and a maximal phase shift of -90° . This opamp does not oscillate for any amplifier. It has a very high DC gain, which reduces towards higher frequencies reaching to 1 at 1 MHz.

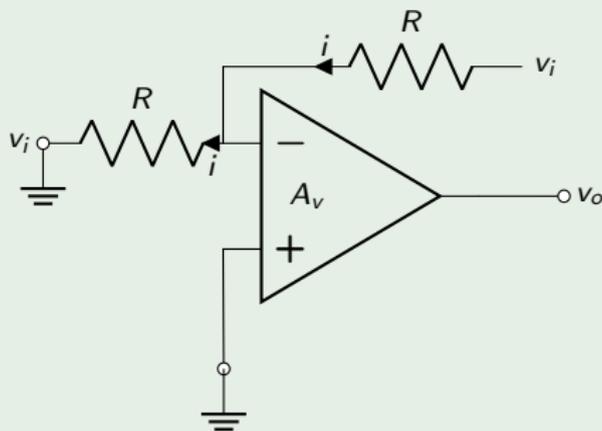
Closed Loop Gain

It might appear that opamp has a poor frequency response since its gain is reduced for frequencies above 40Hz. However, an amplifier is always built using the opamp closed loop gain instead of the open loop. For an amplifier with a gain of 10, the frequency response is flat up to 400kHz and follows the opamp gain thereafter. Closed loop via negative feedback greatly extends the frequency response of the opamp.

Loop Gain

The loop gain of the amplifier circuit is obtained by breaking the feedback loop, injecting a signal, and measuring the gain around the loop which is equal to the opamp gain.

Determine the loop gain of the inverting amplifier whose gain is equal to -1.



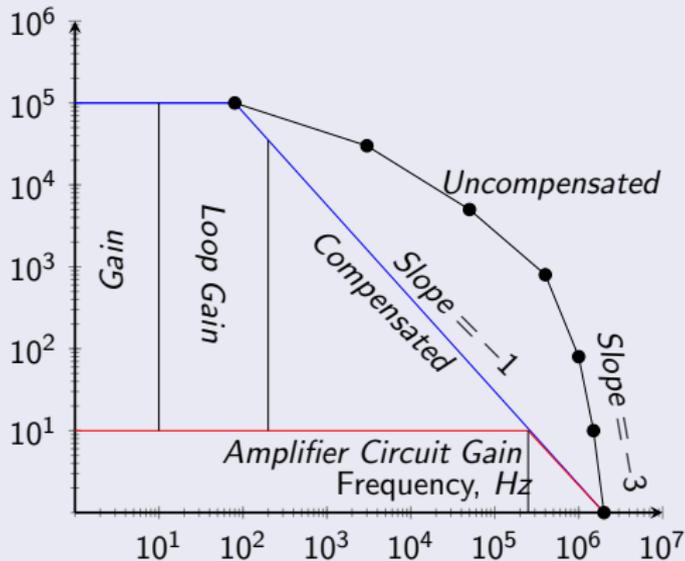
The amplifier circuit input is grounded. The injected signal v_i divided by 2 and amplified by the opamp gain A_v . Thus, the loop gain is equal to $\frac{A_v}{2}$.

At low frequencies, the loop gain is high and the closed loop amplifier circuit behavior is determined by the feedback resistors. At high frequencies, the loop gain is low and amplifier circuit behavior follows the opamp characteristics. High loop gain is good for accuracy and stability.



Gain Bandwidth Product

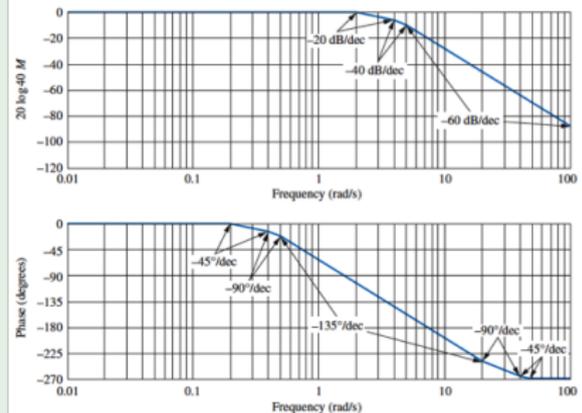
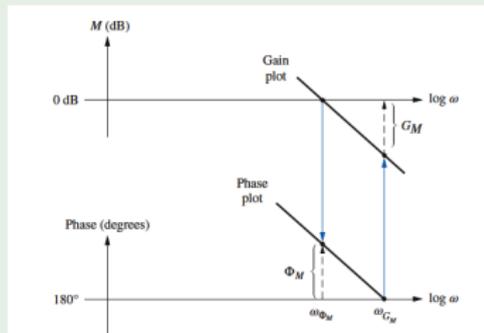
It is equal to the product of gain and bandwidth at a particular frequency. Unity gain bandwidth product is 2MHz.



Gain Margin : change in open loop gain in dB, required at 180° of phase shift to make the closed loop system unstable.

Phase Margin : the change in open loop shift, required at unity gain to make the closed loop system unstable.

Range of Gain for Stability via Bode Plots. $G(j\omega) = \frac{40}{(j\omega+2)(j\omega+4)(j\omega+5)}$. Gain Margin to be 20 dB.



Noise in Electrical Circuits

Potential sources of noise in electronic circuits :

Johnson thermal, flicker and shot noise.

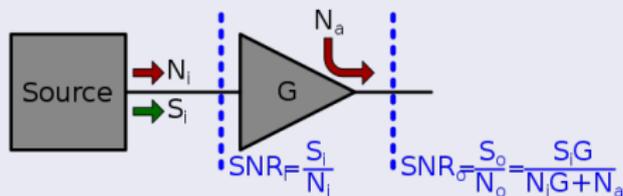
Resistor Voltage $V_{R,noise} = \sqrt{4kTR(BW)}$

Noise Factor $F = \frac{SNR_i}{SNR_o}$

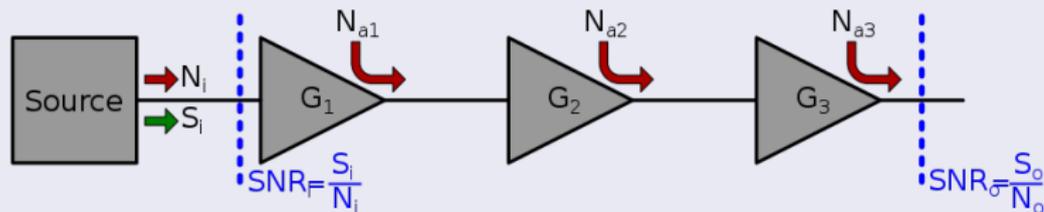
Noise Factor (dB) = $10 \log F$

A system which has a noisy single stage amplifier has

$$F = 1 + \frac{N_a}{G \cdot N_i}$$



Friis Formula



$$N_o = N_i G_1 G_2 G_3 + N_{a1} G_2 G_3 + N_{a2} G_3 + N_{a3}$$

$$SNR_o = S_i G_1 G_2 G_3 / N_o$$

$$F_{Total} = \frac{SNR_i}{SNR_o} = \frac{\frac{S_i}{N_i}}{\frac{S_i G_1 G_2 G_3}{N_i G_1 G_2 G_3 + N_{a1} G_2 G_3 + N_{a2} G_3 + N_{a3}}}$$

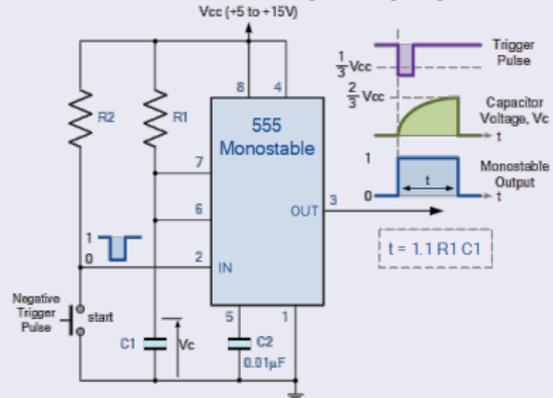
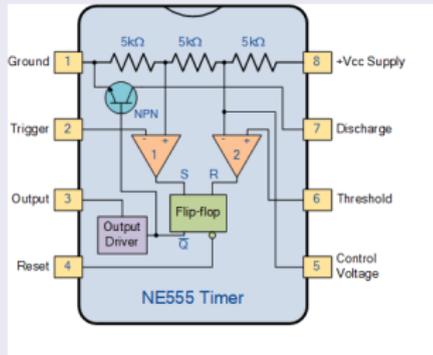
$$= 1 + \frac{N_{a1}}{N_i G_1} + \frac{N_{a2}}{N_i G_1 G_2} + \frac{N_{a3}}{N_i G_1 G_2 G_3}$$

The equivalent noise factor of the entire chain

$$F_{eq} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}}$$

555 Timer Oscillator

Timers are used to generate pulse trains with arbitrary duty cycles.



When a negative (0V) pulse is applied to pin 2, comparator No1 detects this input and sets the state of the flip-flop, changing the output from a LOW to HIGH. This action in turn turns OFF the discharge transistor connected to pin 7, thereby removing the short circuit across the capacitor, C_1 .

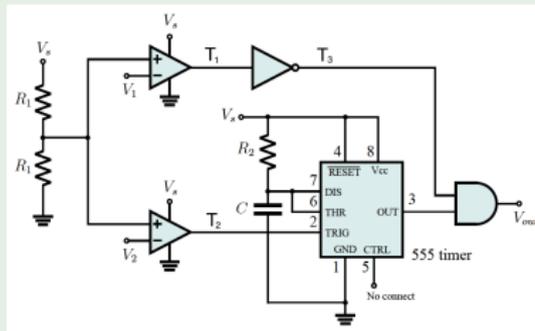
This action allows C_1 to start to charge up through resistor, R_1 until the voltage across the capacitor reaches the threshold (pin 6) voltage of $2/3V_{cc}$. At this point the comparator's output goes "HIGH" and "resets" the flip-flop back to its original state which in turn turns "ON" the transistor and discharges the capacitor to ground through pin 7. This causes the output to change its state back to the original stable LOW value awaiting another trigger pulse to start the timing process over again.

The Monostable 555 Timer circuit triggers on a negative-going pulse applied to pin 2 and this trigger pulse must be much shorter than the output pulse width allowing time for the timing capacitor to charge and then discharge fully. Once triggered, the 555 Monostable will remain in this HIGH unstable output state until the time period set up by the $R_1 C_1$ network has elapsed. The amount of time that the output voltage remains HIGH level, is given by the time constant equation $\tau = \ln(3)R_1 C_1$.

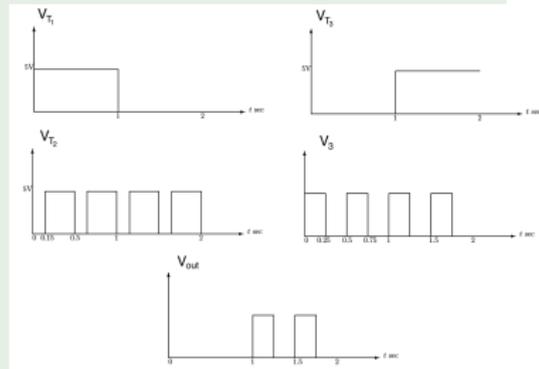
Self Study Question

Consider the following circuit with $V_s = 5\text{ V}$, $R_1 = 1\text{ k}\Omega$, $R_2 = 25\text{ k}\Omega$, and $C = 9.1\text{ }\mu\text{F}$. Two biosensors on a body generate voltage signals V_1 and V_2 . $V_1 = 2.5t$ for $0 \leq t < 2\text{ s}$ and remains at 5 V for $t \geq 2\text{ s}$. V_2 is a square wave with amplitude 2.6 V , period 500 ms and duty cycle 30% .

- 1 Sketch V_1 , V_2 and V_{out} for $0 \leq t < 0\text{ s}$.
- 2 What tolerance is required of the resistance value R_1 in order for this circuit to operate correctly for the given input signals.



$$\tau = \ln(3)R_2C = 25\text{ k} \cdot 9.1\text{ }\mu = 250\text{ ms}$$



V_{out} pulse duration 250ms

Design Problem: Cytotoxicity Measurement

Cytotoxic assays offer an effective means to measure the toxicity of a variety of compounds. These assays are performed by growing cells on a substrate, then exposing them to the compound. If the compound is toxic, the cells will begin to die and undergo *lysis*.

Principle of Measurement: Healthy cell membranes are essentially non conductive, with impedance limited by capacitance. When cells die, their membranes degrade, causing them to leak current and leading to a decrease in impedance, mostly a decrease in resistance.

Using two samples of cells grown on two electrically conducting substrates in a Petri dish: a test sample to be exposed to the unknown compounds, and a control sample shielded from any compounds, design two identical sensors, one for each sample, that couple electrically to the substrate in order to record impedance through the layer of cells to detect if the cells exposed to the compound are dying compared to the control group.

Design specifications:

- For each sensor, inject a current signal to probe the impedance by measuring voltage. Do this separately for the control group of cells, and the test group of cells.
- Don't expose cells to more than $10\ \mu\text{A}$ of current for more than $10\ \mu\text{s}$ at a time for not to kill them.
- This device needs to be portable (powered by batteries), as it will be used in remote locations.
- To save power, the device should only probe/sense cells once every $100\ \text{ms}$.
- The sensor output must be indicated by two LEDs (green and red).
- As long as the resistance of the test group is greater than one half the resistance of the control group, the green LED should remain lit. Otherwise, the red LED should be lit.

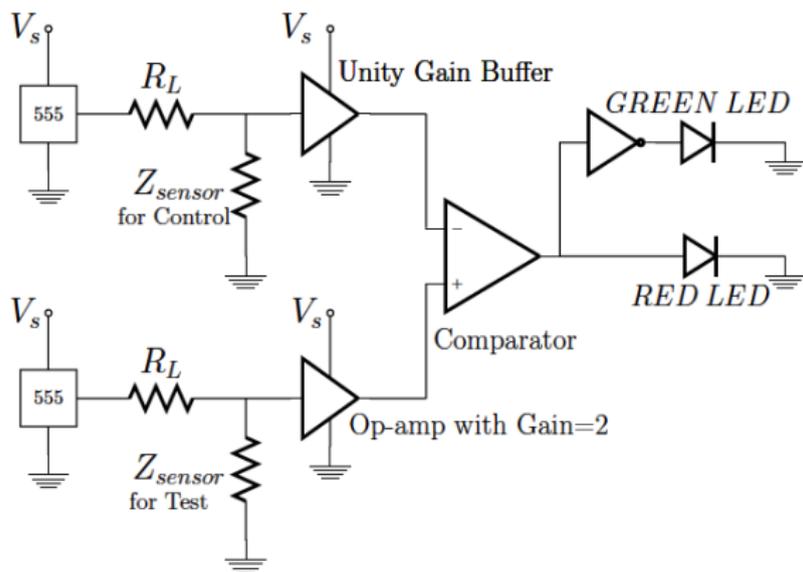


$$T = 100ms$$

$$T_{high} = 10\mu s$$

$$R_L \gg Z_{sensor}$$

$$V_s/R_L = 10\mu A$$



Z_{sensor} for test will decrease as the cells die and the output of the lower opamp will also decrease.